Power corrections in collider processes

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Outline

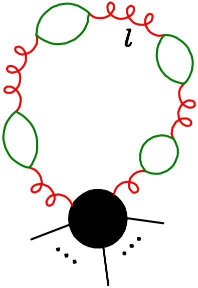
- Power corrections in collider processes
- Renormalons and linear power corrections
- Massless partons
- Massive partons
- e^+e^- annihilation: shape-variables in the 3-jet region.
- Fits to e^+e^- data.

Power corrections for collider processes

- Little is known about power corrections in QCD processes.
- Some simpler processes admit an OPE (the total cross section in e^+e^- annihilation and similar processes, DIS-like processes, B meson decays ...) so that power corrections can be parametrized.
- For the complex collider processes one worries about the presence of linear power corrections, i.e. corrections of the order of Λ/Q, since these could be at the percent level, that is the accuracy one is aiming for at the HL LHC.
- One instrument for the investigation of linear power correction is the study of renormalons in the large b_0 approximation.

ABC of I.R. Renormalons

All-orders contributions to QCD amplitude of the form



$$\int_0^m \mathrm{d}k^p\, \alpha_s(k^2) = \int_0^m \mathrm{d}k^p\, \frac{1}{b_0 \log(k^2/\Lambda^2)}$$

$$= \int_0^m \mathrm{d}k^p \frac{\alpha_s(m^2)}{1 + b_0\alpha_s(m^2)\log\frac{k^2}{m^2}}$$

$$= \alpha_s(m^2)\sum_{n=0}^{\infty}(2b_0\alpha_s(m^2))^n \int_0^m \underbrace{\mathrm{d}k^p \log^n \frac{m}{k}}_{p^n n!}.$$

Asymptotic expansion.

- Minimal term at $n_{\min} \approx \frac{1}{2pb_0\alpha_S(m^2)}$.
- Size of minimal term: $m^p\alpha_s(m^2)\sqrt{2\pi n_{\min}}e^{-n_{\min}} \approx \Lambda^p$.

Large-n_f all-order result

Given an (IR safe) observable O, we introduce the notation

- Φ_B, phase space;
- Φ_g, phase space for the emission of one massive gluon with mass λ,
- $\Phi_{q\bar{q}}$, phase space for the emission of a $q\bar{q}$ pair

the all-order result can be expressed in terms of

- $\sigma_B(\Phi_B)$, the differential cross section for the Born process;
- $\sigma_v(\lambda, \Phi_B)$, the virtual correction to the Born process due to the exchange of a gluon of mass λ;
- The real cross section $\sigma_{g^*}(\lambda, \Phi_{g^*})$, obtained by adding one massive gluon to the Born final state;
- The real cross section $\sigma_{q\bar{q}}(\Phi_{q\bar{q}})$, obtained by adding a $q\bar{q}$ pair, produced by a massless gluon, to the Born final state;

Large-n_f all-order result

Defining:

$$
\begin{aligned}
T_O(\lambda) &= \overbrace{V_O(\lambda) + R_O(\lambda)}^{\text{result for a gluon with mass } \lambda} + \overbrace{\Delta_O(\lambda)}^{\text{Seymour,P.N.1995}}, \\
V_O(\lambda) &= \int \mathrm{d}\Phi_{\mathrm{b}}\, \sigma_{\mathrm{v}}^{(1)}(\lambda^2, \Phi_{\mathrm{b}})\, O(\Phi_{\mathrm{b}}), \\
R_O(\lambda) &= \int \mathrm{d}\Phi_{g^*}\, \sigma_{g^*}^{(1)}(\lambda^2, \Phi_{g^*}) O(\Phi_{g^*}), \\
\Delta_O(\lambda) &= \frac{3\pi\lambda^2}{\alpha_S T_F} \int \mathrm{d}\Phi_{q\bar{q}} R_{q\bar{q}}(\Phi_{q\bar{q}}) \delta(m_{q\bar{q}}^2 - \lambda^2) \left[O(\Phi_{q\bar{q}}) - O(\Phi_{g^*})\right]
\end{aligned}
$$

The Δ term vanishes if the observable is totally inclusive in the radiated partons.

It turns out that a linear term in λ in the expansion of $T(\lambda)$ around zero is associated with linear renormalons.

Large-n_f all-order result

The all-order result is given by

$$\langle O \rangle = B_O - \int \mathrm{d}\lambda \frac{\mathrm{d}T_O(\lambda)}{\mathrm{d}\lambda} \underbrace{\frac{1}{\alpha_S} \overbrace{\left[\frac{1}{\pi b_0} \arctan \frac{\pi b_0 \alpha_S}{1 + b_0 \alpha_S \log \lambda^2/\mu_C^2}\right]}^{\text{Beneke,98}}}_{\alpha_{\mathrm{s,eff}}(\lambda)/\alpha_S}$$

It is easy to show that a linear λ term in $T_O(\lambda)$ leads to a factorial growth related to a linear IR renormalon. In fact

$$\int \mathrm{d}\lambda \left[\frac{1}{\pi} \arctan \frac{\pi b_0 \alpha_S}{1 + b_0 \alpha_S \log \lambda^2/\mu_C^2}\right] = \frac{1}{\pi} P \int_0^\infty \mathrm{d}t \frac{\exp\left(-\frac{t}{2b_0\alpha_S}\right)}{1-t} - \exp\left(-\frac{1}{2b_0\alpha_S}\right) + \text{terms analytic in } \alpha_S. \tag{1}$$

Large-n_f all-order result

- We have a well-defined procedure for the computation of the T function..
- Can be computed semi-numerically. This approach has been followed in
  - Ferrario Ravasio, Oleari, P.N.,2019 for studies related to the top mass measurements.
  - Ferrario Ravasio, Limatola, P.N.,2021 for showing the absence of linear corrections to the p_T spectrum of the Z in hadronic collisions.

Gavin Salam had often shown an argument in favour of the presence of linear power corrections to the inclusive p_T spectrum of the Z boson, based upon the fact that the soft radiation associated to this process is not azimuthally symmetric. Our attemt to actually compute such an effect in a model theory gave negative results.

It is however difficult, numerically, to show the absence of a correction, especially in this case where the cancellation of soft-collinear divergence between the virtual (computed analytically) and real (computed numerically) is involved. Analytic results were found:

- Analitic approach for massless partons:
  Caola,Ferrario Ravasio,Limatola,Melnikov, PN 2021,[2108.08897], same authors + Ozcelik 2022[2204.02247]
- Analitic approach for massive partons:
  Makarov, Melnikov, Ozcelik, PN, 2023, [2302.02729], 2024[2308.05526]

Processes with massless partons

The result is based upon two observations:

- Virtual corrections have no linear power corrections.
  One can show that the virtual integrals give rise to constants, logs and double logs of λ, but no linear terms in λ.
- The real emission term can be written in a factorised form:

$$\mathrm{d}\Phi_g = J \times \mathrm{d}\Phi_B \frac{\mathrm{d}^3 k}{k_0} \tag{2}$$

  through the choice of a mapping to an underlying Born $\Phi_g \leftrightarrow \{\Phi_B, k\}$, (or choice of a recoil scheme).
  It can be shown that if the mapping is linear in k for small k, no linear renormalons are present after the k integration. So: In inclusive cross sections at fixed undelying Born no renormalons are present.

Processes with massless partons

Old and new results can be derived:
Linear corrections are absent in

- DIS (must be the case because of the OPE)
- Drell-Yan total cross section Beneke and Braun
- Drell-Yan rapidity distribution Dasgupta
- Dreal-Yan double differential cross section in transverse momentum and rapidity distribution of the pair (new)
- In e^+e^-, shape variables power corrections can be computed also in the 3-jet regime!
  Before they had been computed only in the 2-jet limits, with the only exception of the C-parameter in the 3-jet symmetric limit (Luisoni,Monni,Salam,2019)

The results on DIS and Drell-Yan follow because on can find an appropriate mapping that also maintains fixed Q^2 and x_{bj} for DIS, and the Drell-Yan pair kinematics for Drell-Yan.

Processes with massive partons

The generic statement that can be made for massless partons cannot be generalized to the massive case. Nevertheless, with a reasoning inspired by the Low-Burnett-Kroll theorem, some results can be obtained also in this case. In particular:

- The absence of linear renormalons can be derived for B meson decays, as long as the B mass is expressed in a short-distance scheme (like the $\overline{\mathrm{MS}}$ one). This result was already obtained by Beneke, and it also follows from the existance of an OPE for includive B decays.
- The absence of linear renormalon in the t-channel, total single top cross section (if m_t is in a short distance scheme!), and the computation of linear corrections in the top differential distributions.
- The absence of linear renormalons in $q\bar{q} \to t\bar{t}$ total cross section (again with m_t in a short distance scheme), and the computation of linear corrections in the top differential distributions.

Single Top

The result for the differential distribution can be expressed as a shifts in the argument of the Born cross section. For the transverse momentum and rapidity of the top the shift are given by

$$\begin{aligned}\frac{\delta_{\rm NP}[p_\perp]}{p_\perp} &= \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t} \\ \delta_{\rm NP}[y_t] &= \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t} \times \frac{8m_t^2\, s\, ch^2(y_t)}{(s+m_t^2)^2}\end{aligned}$$

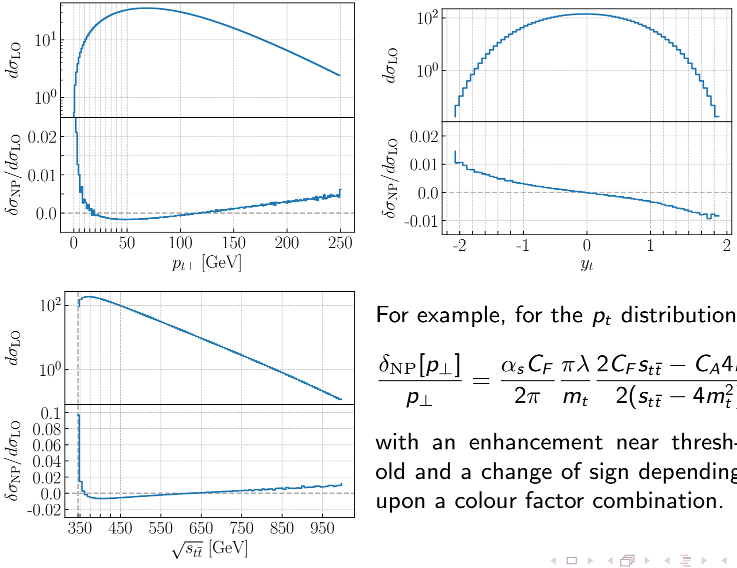
Since we have

$$\frac{\delta_{\rm NP} m_t}{m_t} = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}$$

we can use current determination of the top quark pole mass renormalon uncertainty $\delta_{\rm NP} m_t = 0.1 - 0.2$ GeV to estimate these effects.

$q\bar{q} \to t\bar{t}$

The results have a more interesting structure



For example, for the p_t distribution

$$\frac{\delta_{\rm NP}[p_\perp]}{p_\perp} = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\frac{2C_F s_{t\bar{t}} - C_A 4m_t^2}{2(s_{t\bar{t}} - 4m_t^2)},$$

with an enhancement near threshold and a change of sign depending upon a colour factor combination.

e^+e^- shape variables: why we can compute them

Recall our results for massless partons, applied to the process $e^+e^- \to q\bar{q}\gamma$. We can write, for a generic shape variable

$$S = \int \mathrm{d}\sigma(p,k)\,[S(p,k) - S(\tilde{p})] + \int \mathrm{d}\sigma(p,k) S(\tilde{p}),$$

where $\tilde{p}$ are the underlying Born momenta. According to our finding the second term cannot yield linear NP corrections, since it can be integrated over the radiation variables at fixed underlying Born.
So, only the first term is left, and it receives contributions only from the real emission of a soft gluon, that, thanks to the suppression of the square bracket, can be computed in the soft limit.

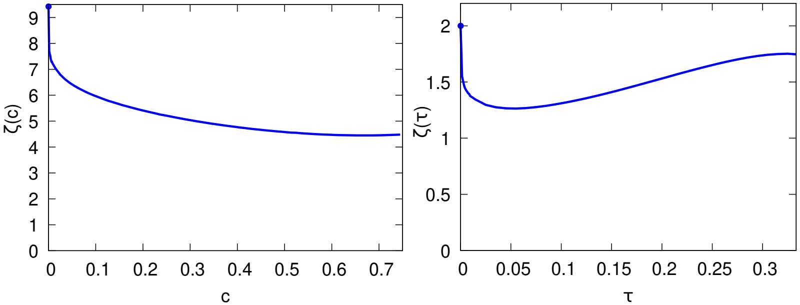
Since the correction depends only upon the soft eikonal approximation for the soft emission, we can put forward the hypothesis that it can be extended to final states involving also gluons.

Complications due to the gluon splitting into a $q\bar{q}$ give rise to the same Milan factor Dokshitzer, Marchesini and Salam that is found in the case of the 2-jet region.

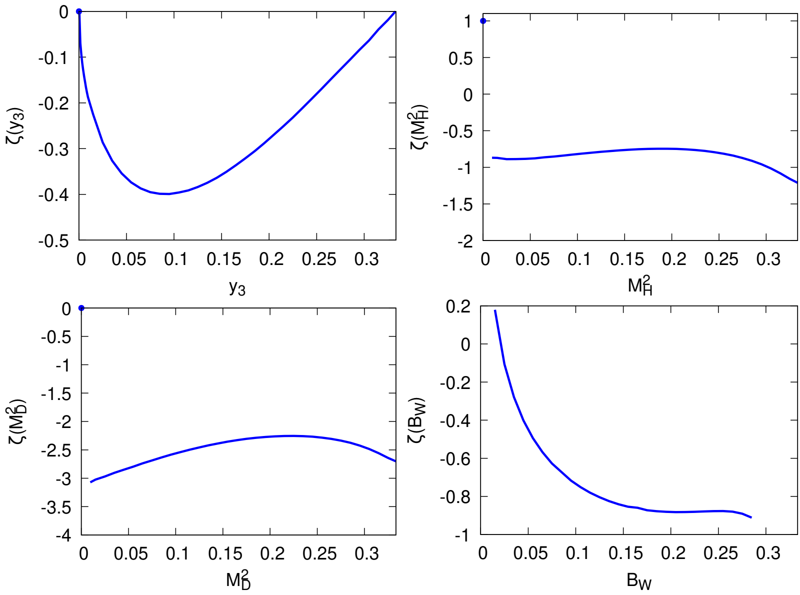
Non-perturbative corrections can be parametrized as a shift in the perturbative cumulant distribution:

$$\Sigma(s) \longrightarrow \Sigma(s + H_{\mathrm{NP}}\zeta(s)), \quad \text{where} \quad \Sigma(s) = \int \mathrm{d}\sigma(\Phi)\theta(s - s(\Phi))$$

and $H_{\mathrm{NP}} \approx \Lambda/Q$ is a non-perturbative parameter that must be fitted to data.



The dot in the plots represents the constant value that was used in earlier studies. The value of $\zeta(c)$ at the symmetric point $c = 3/4$ was also computed by Luisoni,Monni,Salam 2021.



(G.Zanderighi,P.N.2023) In some cases ζ is negative!

α_s from e^+e^- shape variables

- Historically the framework of choice to measure α_s directly from the $q\bar{q}g$ vertex.
- In practice: very convincing at the 10% level; affected by non-perturbative uncertainties if one wants higher precision
- $\alpha_s(M_Z)$ from NNLO+NLL+Monte Carlo models:
  - 0.1224 ± 0.0039 ALEPH 2009, [arXiv:0906.3436].)
  - 0.1189 ± 0.0043 OPAL 2011, [arXiv:1101.1470])
  - 0.1172 ± 0.0051 JADE 2009, [arXiv:0810.1389]

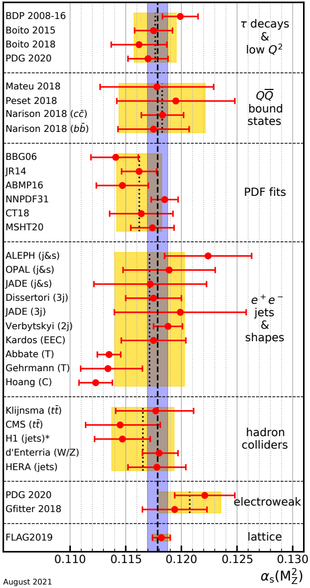
The use of Monte Carlo models to correct for hadronization effects have long been criticized, since the interplay of perturbative and non-perturbative effects in Shower Monte Carlo is not fully clear.

α_s from e^+e^- shape variables

As an alternative, another class of determinations is based upon analytic modeling of non-perturbative effects, using methods like SCET, dispersive models and low scale QCD effective couplings, and using NNLO+N^{3}LL calculations:

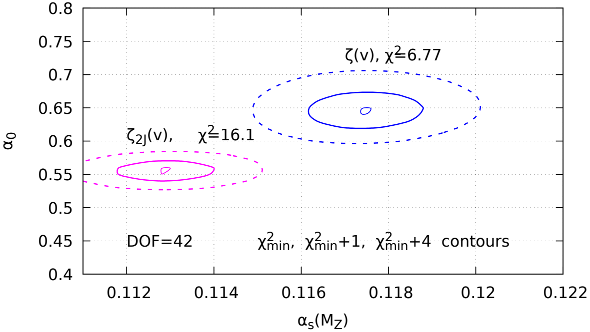
- 0.1135 ± 0.0011 R.Abbate *et al*, 2011, [arXiv:0809.3326]
- $0.1134 \ {}^{+0.0031}_{-0.0025}$ Gehrmann,Luisoni,Monni, 2013,[arXiv:1210.6945]
- 0.1123 ± 0.0015 Hoang *et al*, 2015 [arXiv:1501.04111]

They tend to result in a rather low value, not in good agreement with world data.



Results from Zanderighi, P.N. 2023

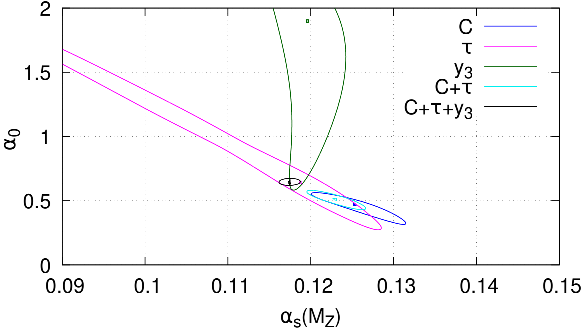
Simultaneous fit to C, t and y_3, both for our newly computed $\zeta(v)$, and, for comparison, with $\zeta(v) \to \zeta_{2J}(v) = \zeta(0)$ (traditional method for the computation of power corrections).



The central value is at $\alpha_s(M_Z) = 0.1174$, $\alpha_0 = 0.64$. The "traditional" method leads to smaller values of α_s.

Results from Zanderighi, P.N. 2023

Individual fits:



Only the combination of the three observables leads to a sensible determination of α_s

VERY PRELIMMINARY: followup on e^+e^- fits

(with G. Zanderighi)

- We want to include all data, also at different energies.
- We want a better treatment of theoretical errors, by performing the fits at the central scale, and at its variations by a factor of 2 below and above.
- We hope that the availability of different energies should lead to a more reasonable determination for single observables.
- In order to get a better fit of the very precise Z-peak data, we chose the central scale to be a function of the shape variable:
  We first compute the average k_T as a function of the value of each shape variable (computed at the LO level), and then choose the k_T as central value of the scale. Fitting only ALEPH data on the Z peak we get:

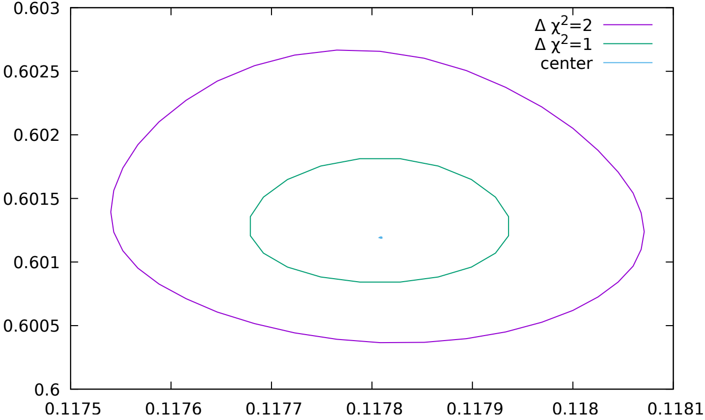
	χ^2/dof	$\alpha_S(M_Z)$	α_0
fixed scale	1.99	0.1157	0.61
running scale	1.21	0.1183	0.61

EXPERIMENTS AND ENERGIES

DELPHI 91.2 45 66 76 133 161 172 183 189 192 196 200 202 205 207
ALEPH 91.2 133 161 172 183 189 200 206
OPAL 91.2 133 177 197
L3 91.2 41.4 55.3 65.4 75.7 82.3 85.1 130.1
136.1 161.3 172.3 182.8 188.6 194.4 200
JADE 22 35 44
TRISTAN 58
JADEOPAL 91.2 35 44 133 161 172 183 189
SLD 91.2

VERY PRELIMINARY PLOTS

Fitting Thrust, C-parameter and y_3 at the same time



Leading to $\alpha_s(M_Z) = 0.1178$, and $\alpha_0 = 0.60$, with $\chi^2 = 1257.7$ over 947 degrees of freedom ($\chi^2/\mathrm{dof} = 1.33$).

VERY PRELIMINARY PLOTS

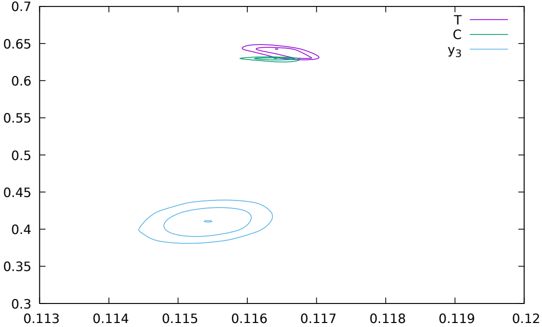
Results for scale variation (scfac), and variation lower limit (llfac). The lower limit is obtained by multiplying the peak position by llfac, with some further adjustments for y_3.

scfac	llfac	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/dof	ndeg
0.5	1.5	0.1159	0.6969	2263	2.65	852
0.5	1.7	0.1159	0.6944	2213	2.64	837
0.5	2	0.1168	0.6671	1961	2.40	816
1	1.5	0.1164	0.6267	1631	1.65	983
1	1.7	0.1178	0.6029	1257	1.33	943
1	2	0.1182	0.5875	1147	1.28	895
2	1.7	0.1171	0.5725	1619	1.68	964
2	2	0.1168	0.5829	1492	1.63	915

Notice that the best fit is for our central scale, as long as the low limit is not too small.

VERY PRELIMINARY PLOTS

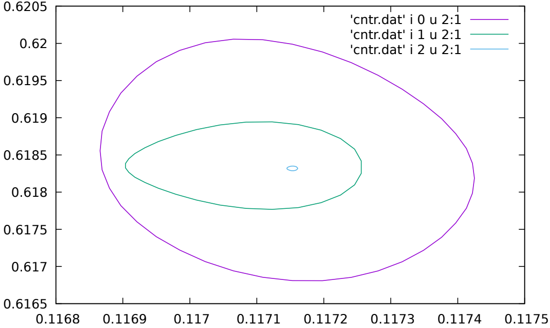
Fitting Thrust, C-parameter and y_3 individually:



we see a tension between y_3 and the C/T results, especially regarding the value of α_0. On the other hand the contribution to the χ^2 from y_3 alone in the global fit shown earlier is $1 \times \mathrm{ndeg}$

VERY PRELIMINARY PLOTS

Fitting C and T together:



with $\alpha_s(M_Z) = 0.1171$ and $\alpha_0 = 0.6185$, $\chi^2/\mathrm{dof} = 1.33$.

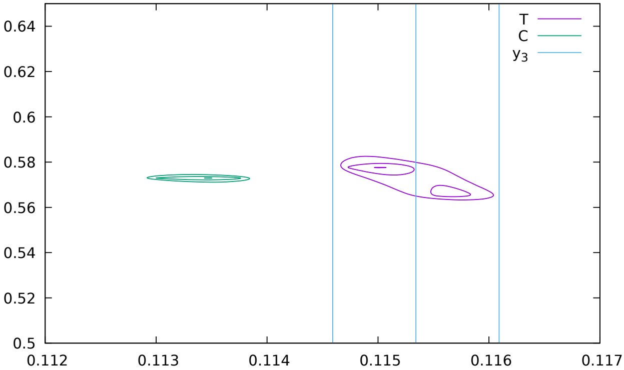
Conclusions

- I illustrated recent progress on the calculation of non-perturbative corrections in the large β_0 limit
- Very simple results for a number of observables involving only massless coloured partons.
- More complex results when heavy quarks are present.
- The simplest framework where these results can be tested is the study of shape variables in e^+e^- annihilation. A previous attempt used only ALEPH data at M_Z.
- Ongoing, encouraging results using all available energy and experiments. The fit to y_3 raises some doubts.
- To be done: better understanding of y_3, and how to estimate theoretical errors.

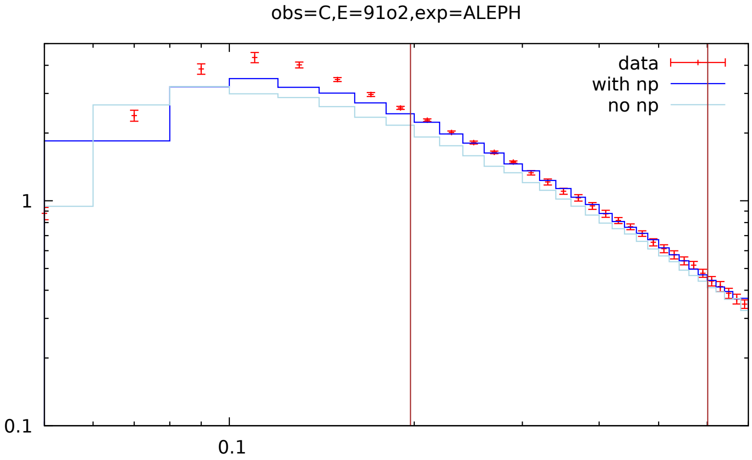
BACKUP SLIDES

(old) NP effects

Taking the ζ functions to be constant, equal to thei two-jet value:



Quality of the fit



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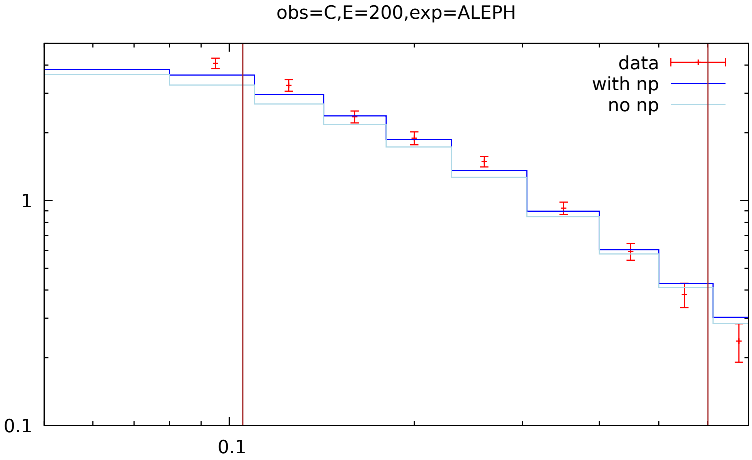

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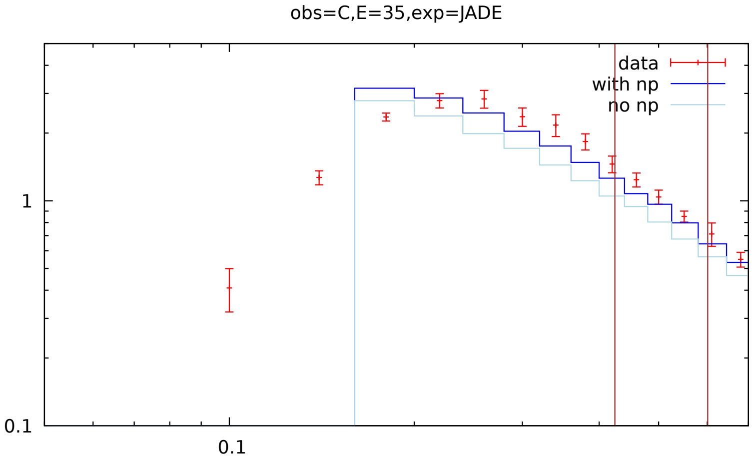

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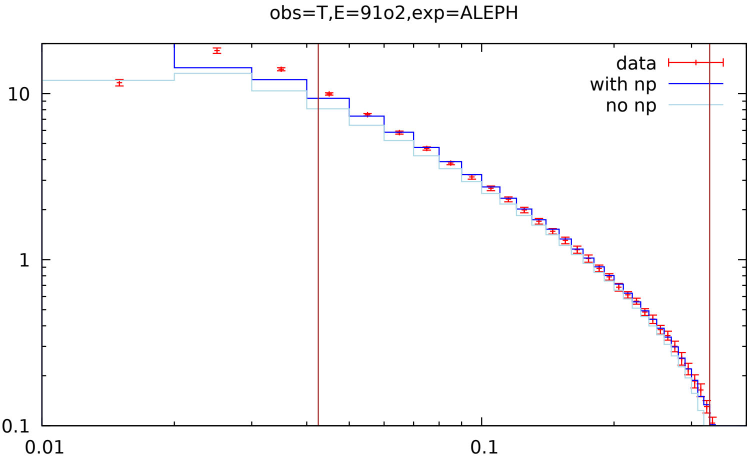

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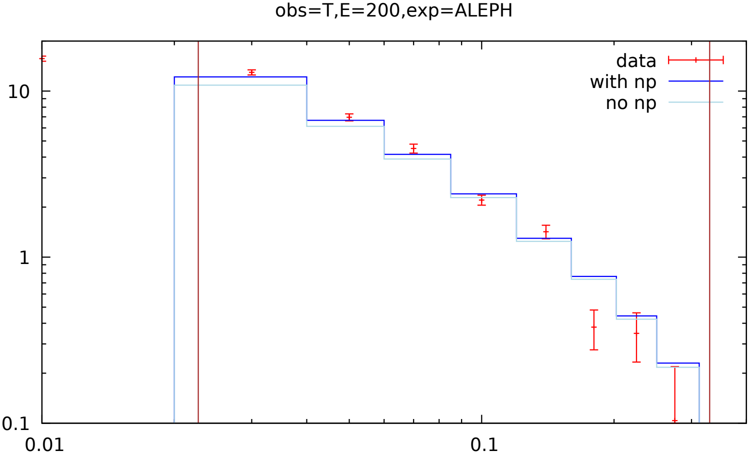

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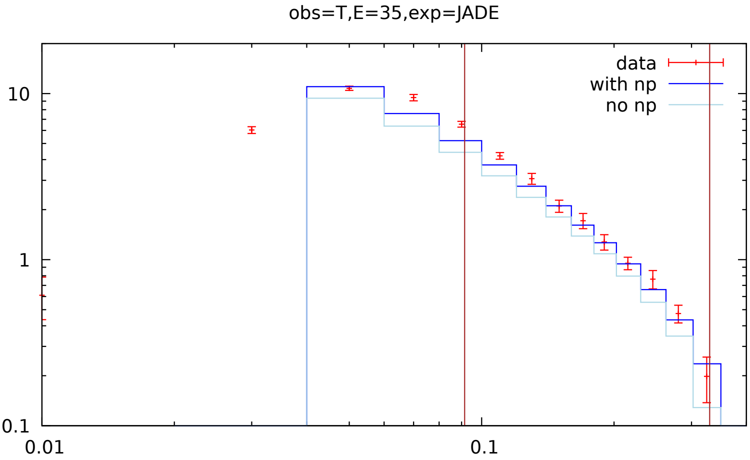
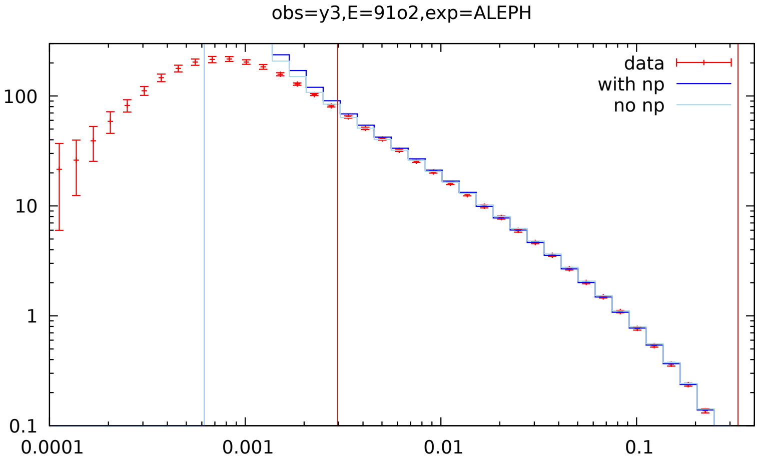
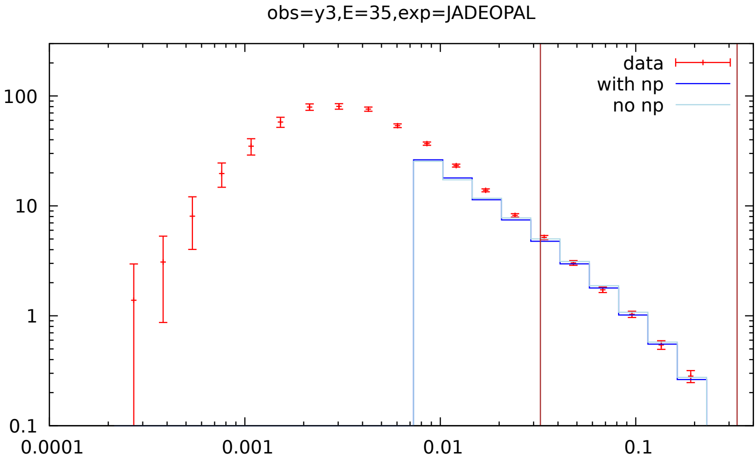


Quality of the fit



Quality of the fit



Fit details

Take v_i to span all bins of all shape variables considered; we define

$$\chi^2 = \sum_{ij} \Delta_i V_{ij}^{-1} \Delta_j, \quad \Delta_i = \left(\frac{1}{\sigma_{\mathrm{exp}}} \frac{\mathrm{d}\sigma_{\mathrm{exp}}(v_i)}{\mathrm{d}v_i} - \frac{1}{\sigma_{\mathrm{th}}} \frac{\mathrm{d}\sigma_{\mathrm{th}}(v_i)}{\mathrm{d}v_i} \right),$$

$$V_{ij} = \delta_{ij}(R_i^2 + T_i^2) + (1-\delta_{ij}) C_{ij} R_i R_j + \mathrm{Cov}_{ij}^{(\mathrm{Syst})}$$

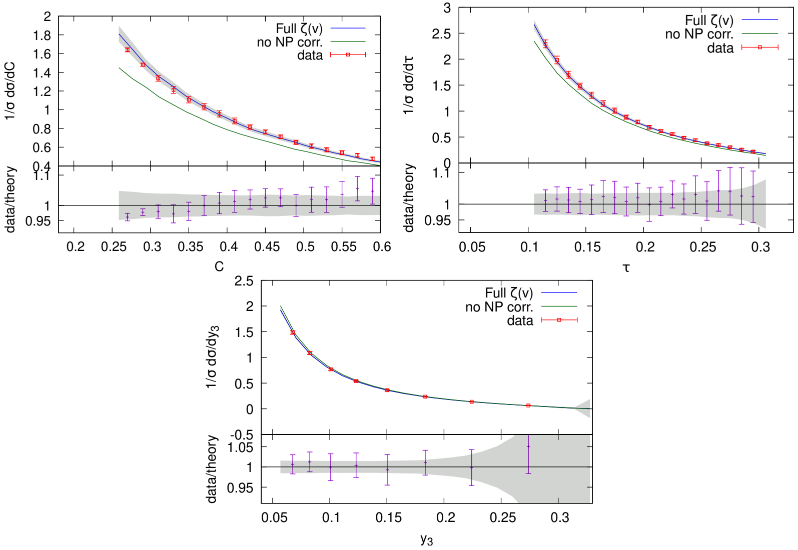
- R_i: statistical error
- T_i: theoretical error (scale variation plus error estimate of non-perturbative shift).
- C_{ij} statistical correlation (from Monte Carlo simulation)
- $\mathrm{Cov}_{ij}^{(\mathrm{Syst})}$: systematics covariance matrix

Variation	$\alpha_s(M_Z)$	α_0	χ^2	$\frac{\chi^2}{N_{\mathrm{deg}}}$
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Ren. sc. $Q/4$	0.1180	0.60	6.1	0.14
Ren. sc. Q	0.1182	0.68	7.9	0.18
NP sch. (b)	0.1186	0.79	6.4	0.15
NP sch. (c)	0.1194	0.84	4.7	0.11
NP sch. (d)	0.1184	0.66	5.2	0.12
P-scheme	0.1150	0.63	9.5	0.22
D-scheme	0.1188	0.79	5.1	0.12
Std. scheme	0.1168	0.58	8.1	0.18
No hq corr.	0.1176	0.68	6.2	0.14
`Herwig 6`	0.1174	0.60	14.7	0.33
`Herwig 7`	0.1174	0.60	10.9	0.25
Ranges (2)	0.1166	0.62	12.3	0.22
Ranges (3)	0.1178	0.69	2.4	0.07
Alt. correl.	0.1180	0.62	5.8	0.13
y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.0	0.00
C, τ	0.1230	0.51	2.0	0.05

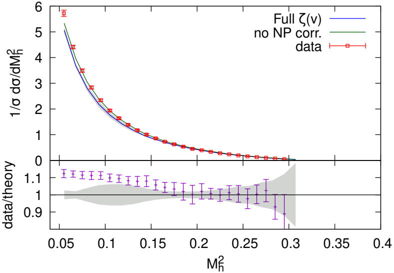
Several variations of setup parameters/methods lead to variations of the central value of order 1%. Among them

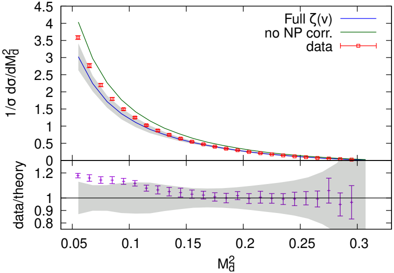
- Central ren. scale
- Ambiguity in implementation of NP corrections
- Treatment of correlation in systematic errors
- Treatment of hadron masses (P, D and std. schemes)

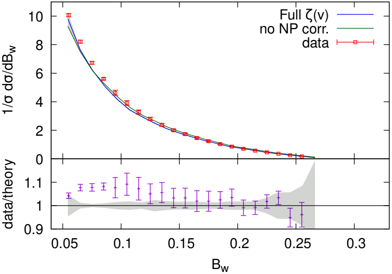
Quality of the fit for C, τ and y_3, using the new calculation of the non-perturbative effect (i.e. the full $\zeta(v)$ dependence.)



Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

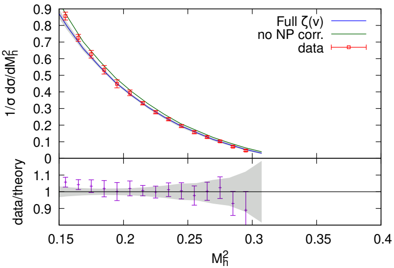


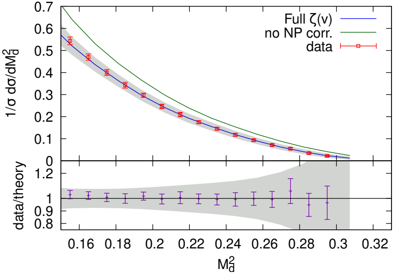


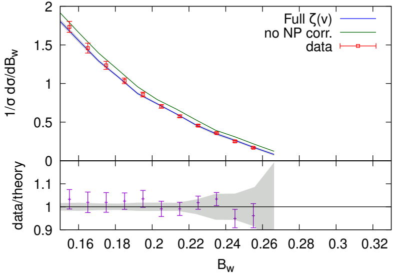


Good fit away from the two jet region.

Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

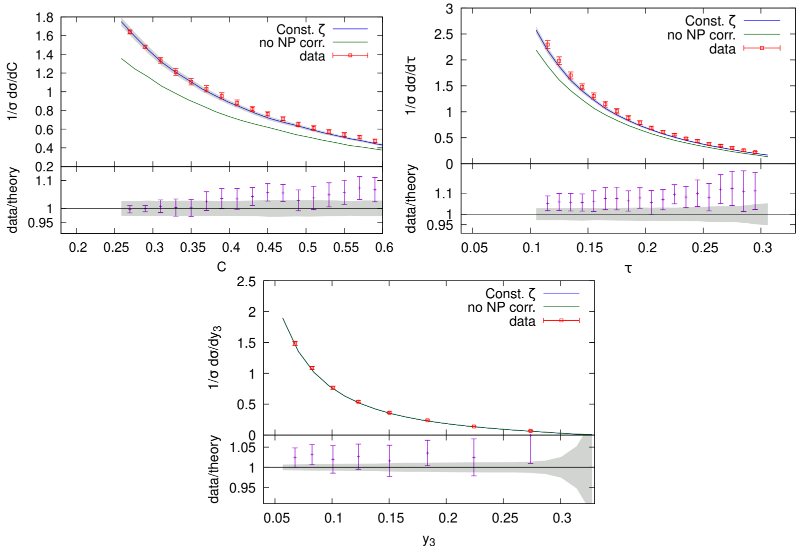




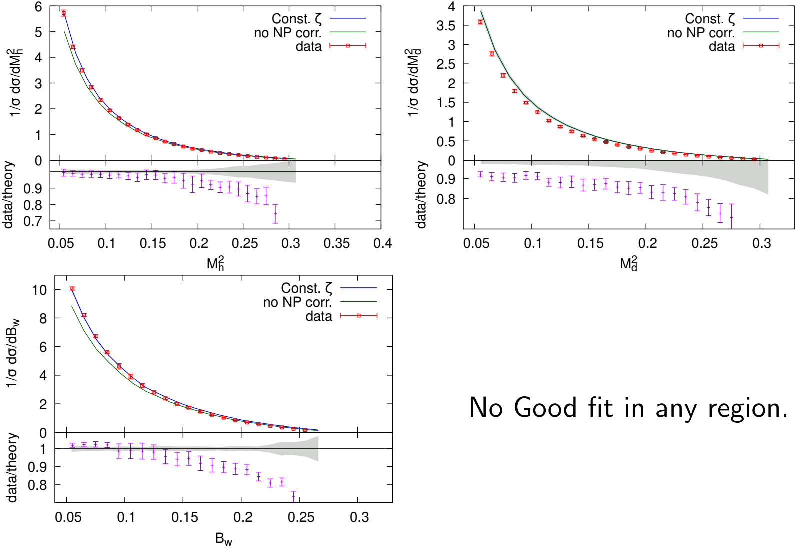


Good fit far away from the two jet region.

Quality of the fit for C, τ and y_3, obtained setting $\zeta(v) = \zeta(0)$.

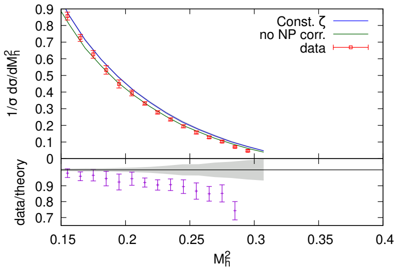


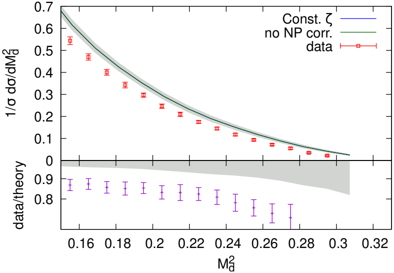
Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.

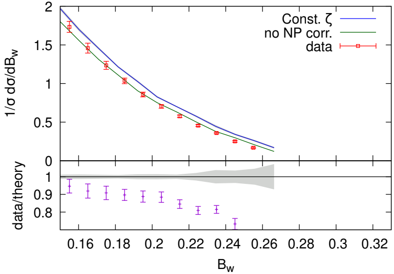


No Good fit in any region.

Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.







No Good fit in any region.