

Recent Progress on PDFs and N3LO PDFs

Ringberg 2024



Nikhef

α_s from Z pT

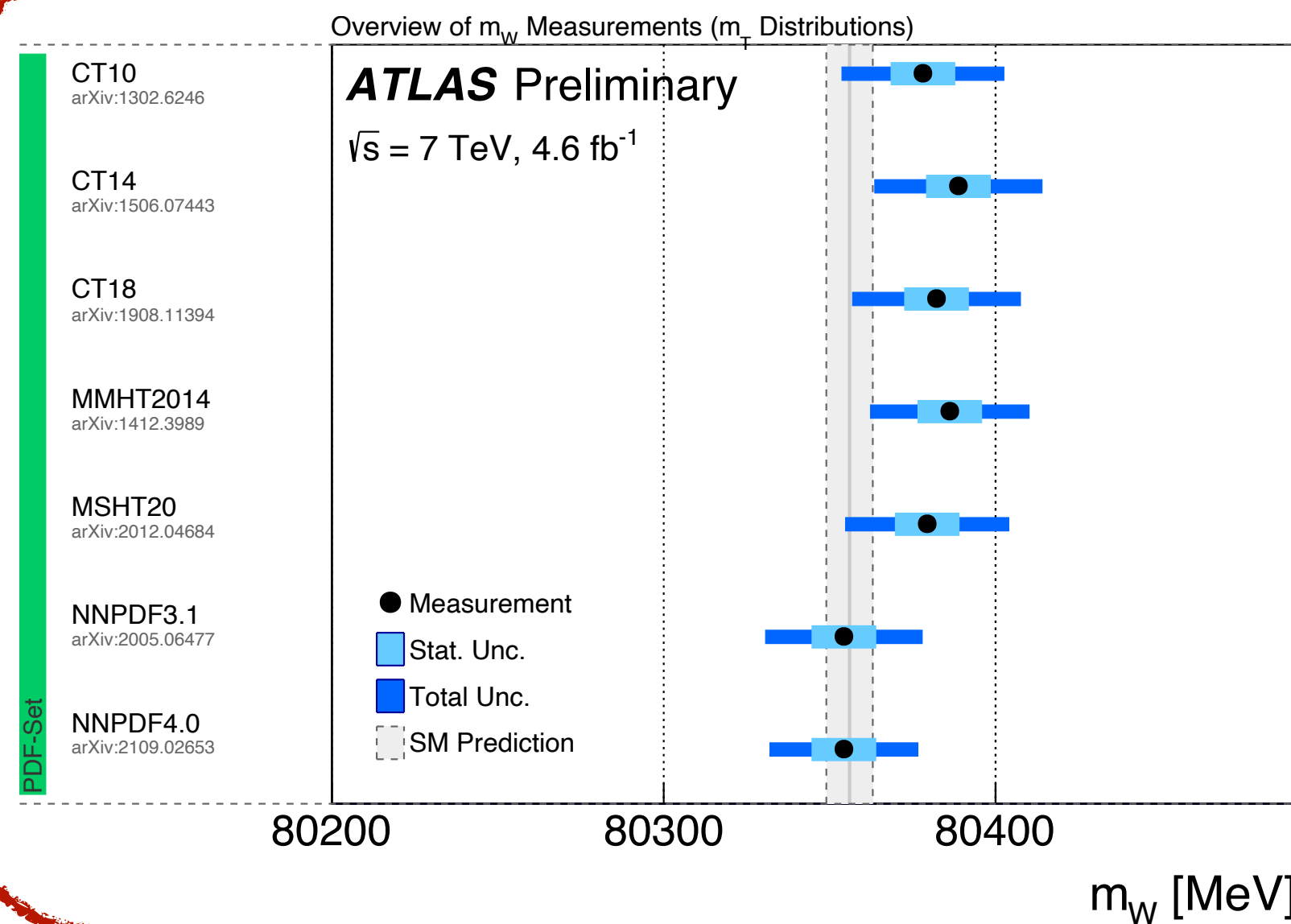
[arXiv:2309.12986](https://arxiv.org/abs/2309.12986)

$$\alpha_s(m_Z) = 0.11847 + 0.00091 - 0.00088$$

$\sim 0.76\%$

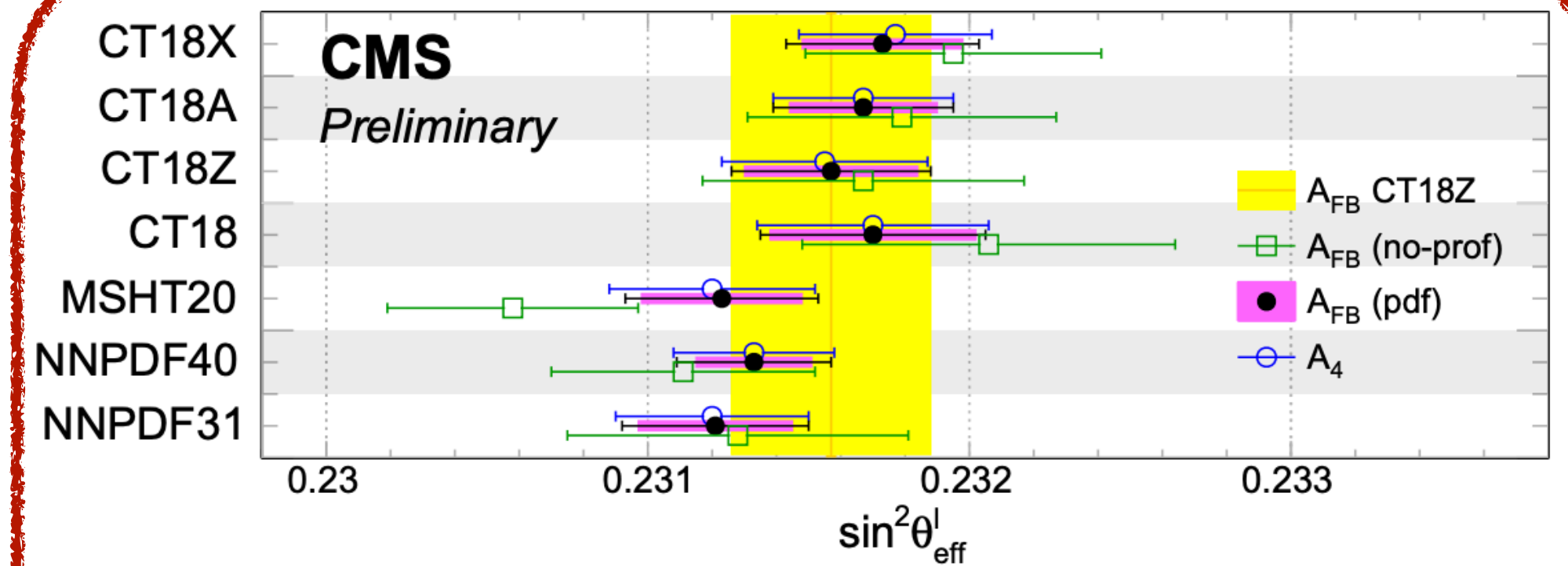
PDF set	$\alpha_s(m_Z)$	PDF uncertainty	$g [GeV^2]$	$q [GeV^4]$
MSHT20 [37]	0.11839	0.00040	0.44	-0.07
NNPDF4.0 [84]	0.11779	0.00024	0.50	-0.08
CT18A [29]	0.11982	0.00050	0.36	-0.03
HERAPDF2.0 [65]	0.11890	0.00027	0.40	-0.04

$\sim 1.7\%$



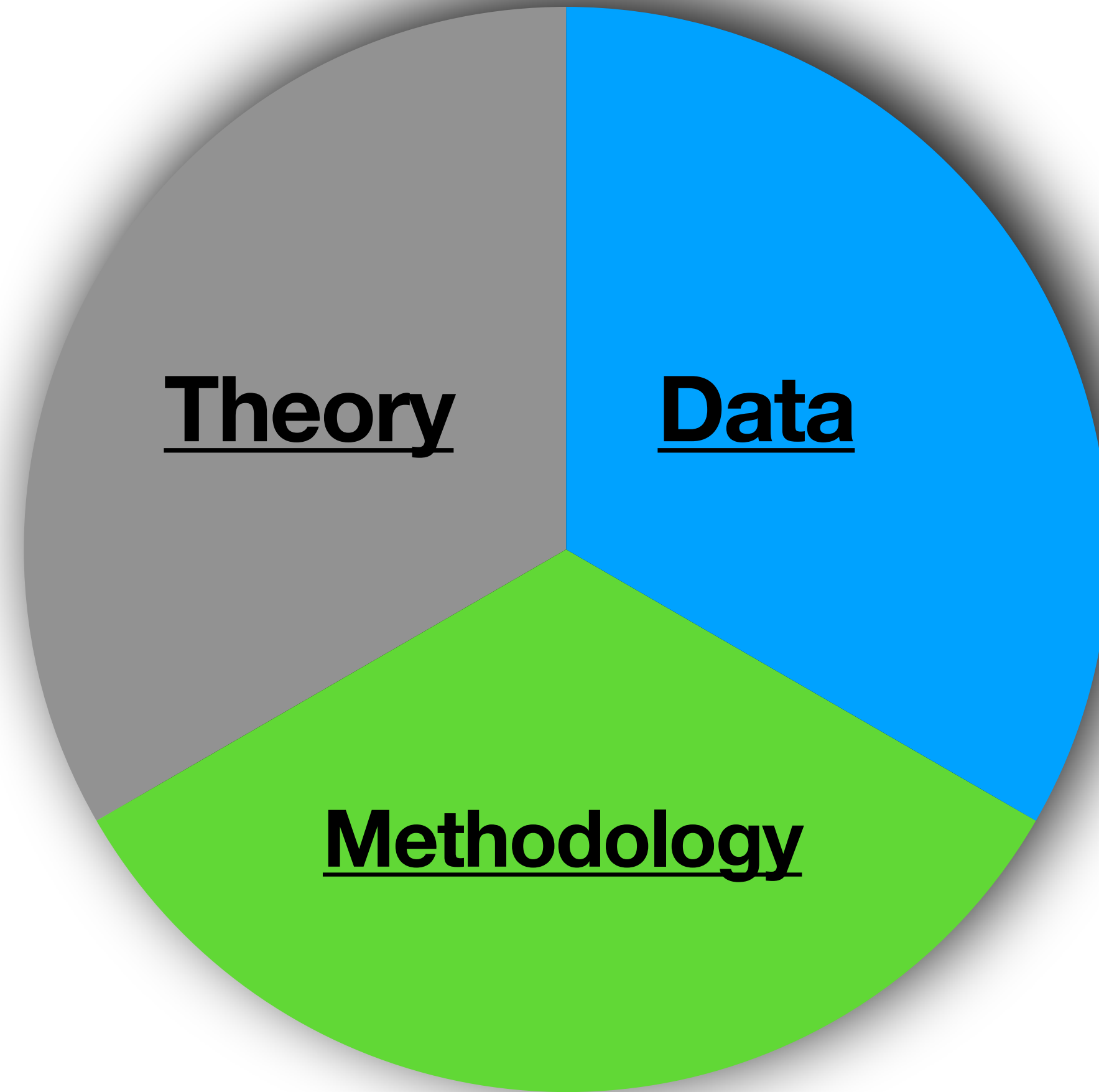
W mass determination

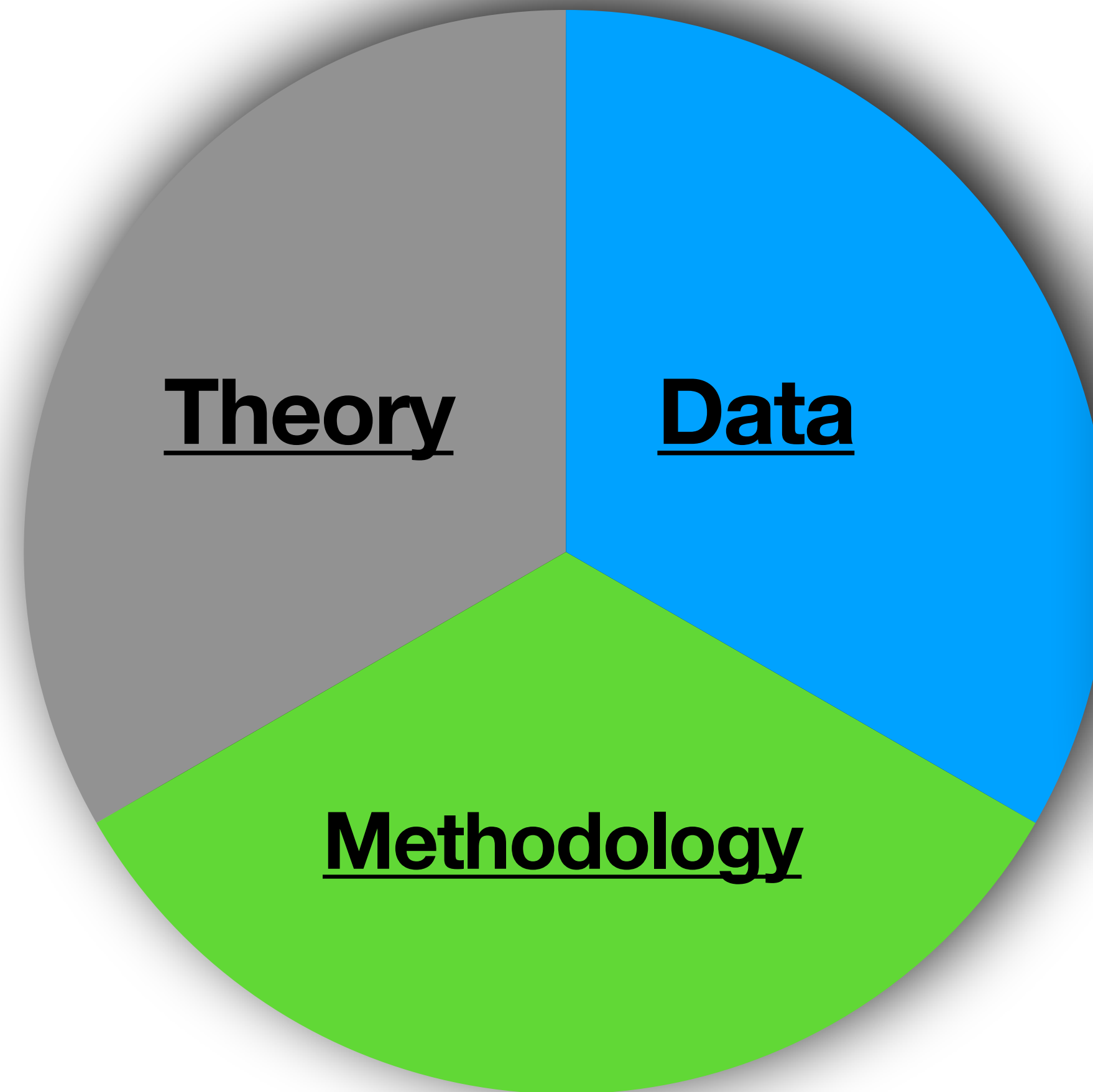
[ATLAS-CONF-2023-004](#)



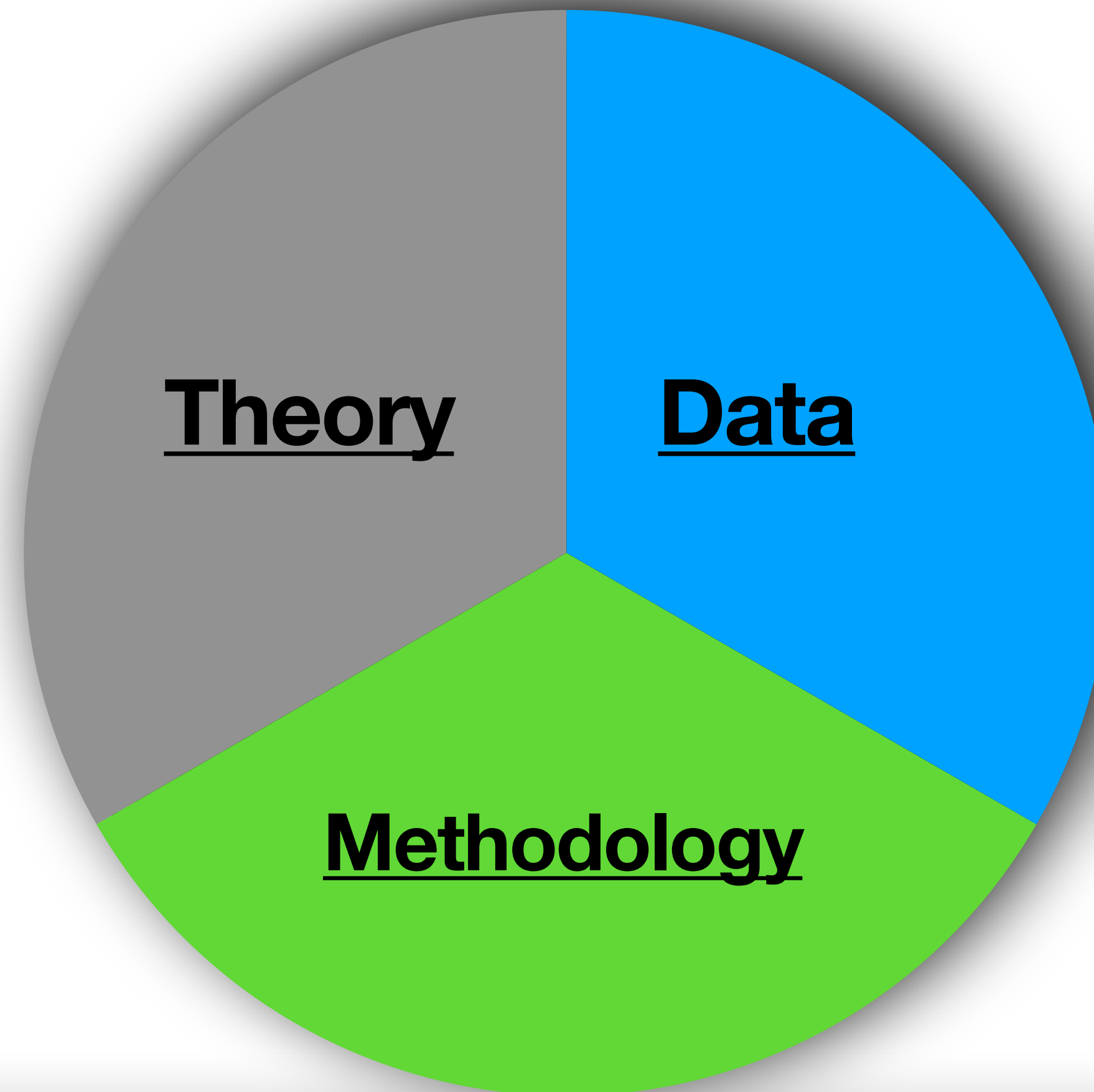
weak mixing angle at 13 TeV

[CMS-PAS-SMP-22-010](#)





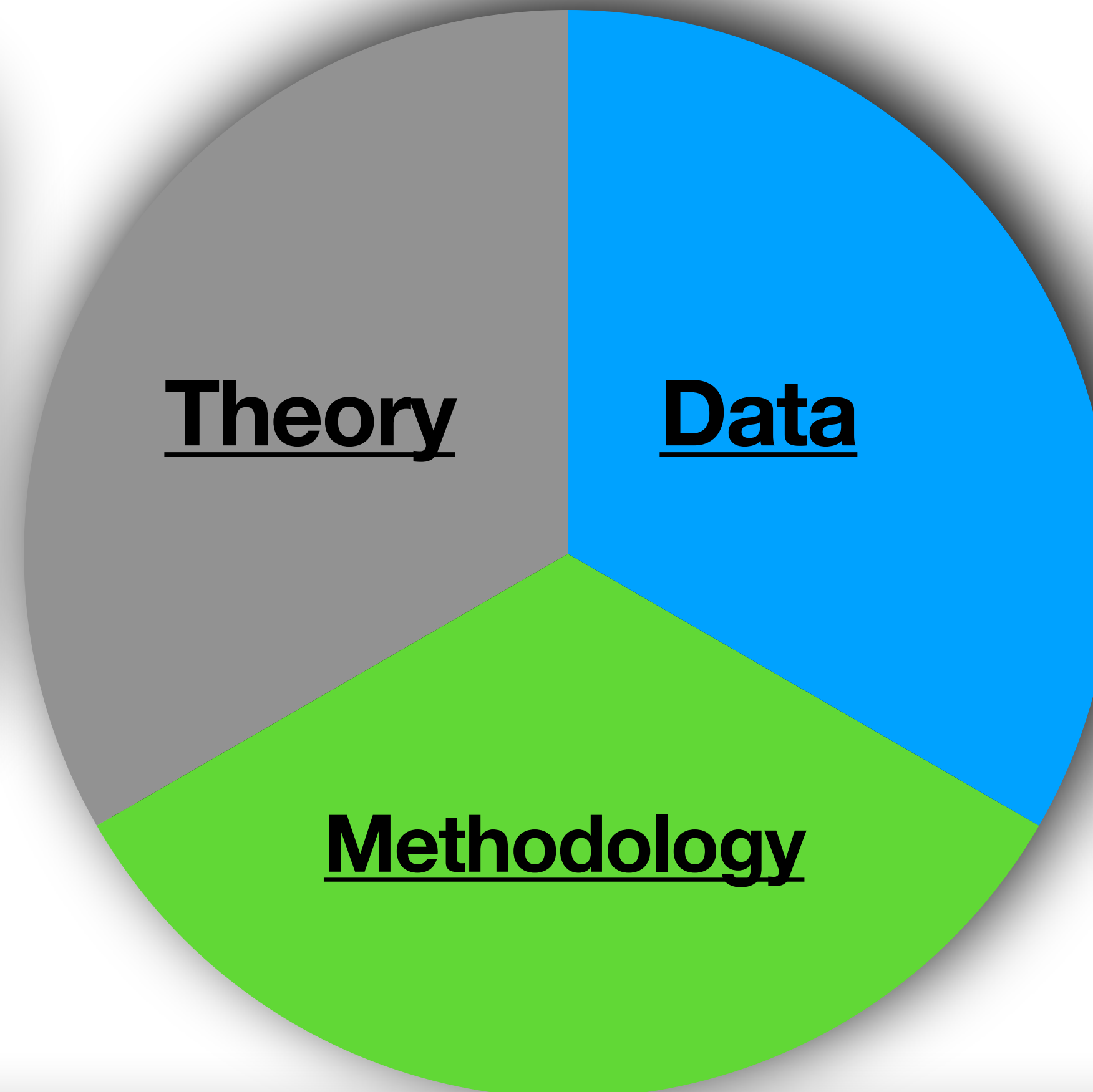
- Impact of jets vs dijets at N3LO [arXiv:2312.12505]
- Impact of 13 TeV $t\bar{t}$ data [PRD 109 (2024)]
- Impact of future data (HL-LHC [Eur. Phys. J. C (2018) 78], EIC [PRD 103 (2021) 096005], FPF [arXiv:2309.09581])



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- Nonparametric regression [arXiv:2404.02964]
- Closure test [EPJC 82 (2022) 4, Talk by Lucian Harland-Lang, DIS2024]

- aN3LO [EPJC 83, arXiv:2402.18635]
- MHOU [arXiv:2401.10319]
- QED [arXiv:2401.08749]
- QED + aN3LO [arXiv:2404.02964]

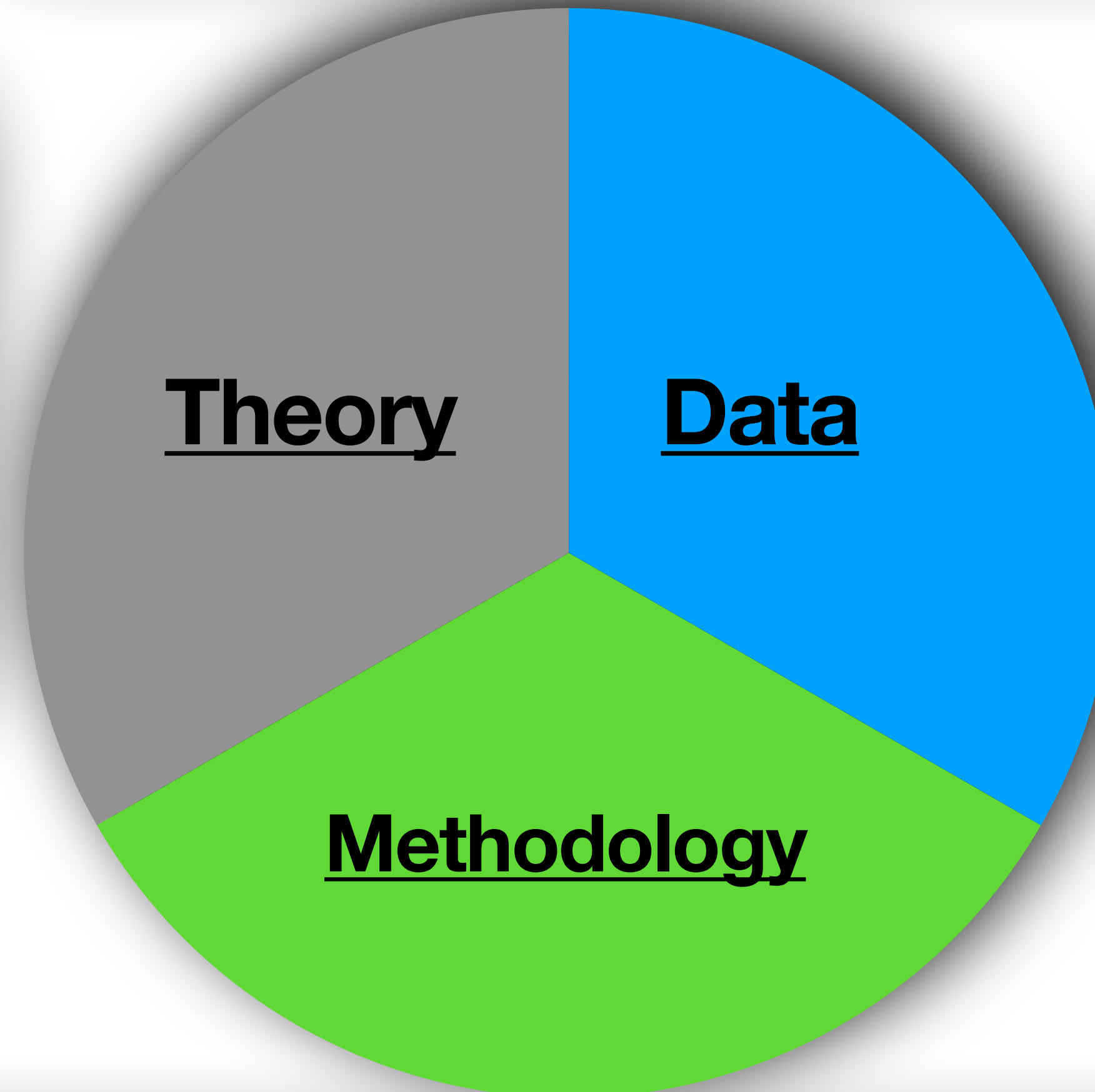


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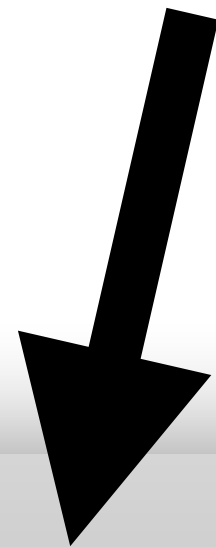
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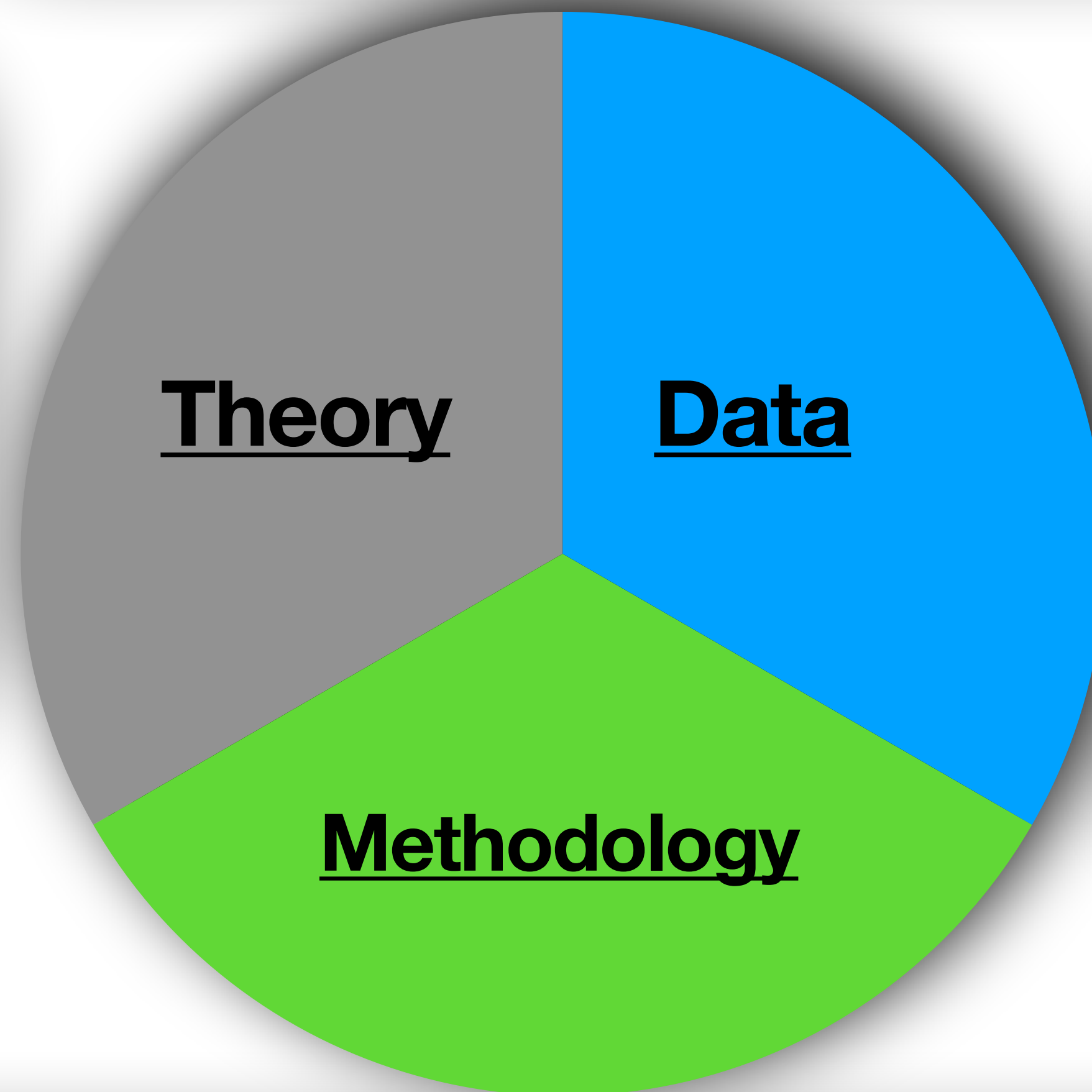
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approximated N³LO PDFs (aN³LO)

Eur. Phys. J. C (2023) 83:185
https://doi.org/10.1140/epjc/s10052-023-11236-0

THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article - Theoretical Physics

Approximate N³LO parton distribution functions with theoretical uncertainties: MSHT20aN³LO PDFs

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Received: 8 August 2022 / Accepted: 19 January 2023
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Abstract We present the first global analysis of parton distribution functions (PDFs) at approximate N³LO in the strong coupling constant α_s , extending beyond the current highest NNLO achieved in PDF fits. To achieve this, we present a general formalism for the inclusion of theoretical uncertainties associated with the perturbative expansion in the strong coupling. We demonstrate how using the currently available knowledge surrounding the next highest order (N³LO) in α_s can provide consistent, justifiable and explainable approximate N³LO (aN³LO) PDFs. This includes estimates for uncertainties due the currently unknown N³LO ingredients,

2	Theoretical procedures
2.1	Hessian method with nuisance parameters
2.2	Multiple theory parameters
2.3	Decorrelated parameters
3	Structure functions at N ³ LO
	$F_{2,q}$
	$F_{2,H}$
4	N ³ LO splitting functions
4.1	Approximation framework: discrete moments
4.2	4-Loop approximations
	$P_{qq}^{(3)}$

The Path to N³LO Parton Distributions

The NNPDF Collaboration:

Richard D. Ball¹, Andrea Barontini², Alessandro Candido^{2,3}, Stefano Carrazza², Juan Cruz-Martinez³,
Luigi Del Debbio¹, Stefano Forte², Tommaso Giani^{4,5}, Felix Hekhorn^{2,6,7}, Zahari Kassabov⁸,
Niccolò Laurenti², Giacomo Magni^{4,5}, Emanuele R. Nocera⁹, Tanjona R. Rabemananjara^{4,5}, Juan Rojo^{4,5},
Christopher Schwan¹⁰, Roy Stegeman¹, and Maria Ubiali⁸

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INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

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INFN, Sezione di Torino, Via Pietro Giuria 1, I-10125 Torino, Italy

¹⁰Universität Würzburg, Institut für Theoretische Physik und Astrophysik, 97074 Würzburg, Germany

This paper is dedicated to the memory of Stefano Catani,
Grand Master of QCD, great scientist and human being

Abstract

We extend the existing leading (LO), next-to-leading (NLO), and next-to-next-to-leading order (NNLO) NNPDF4.0 sets of parton distribution functions (PDFs) to approximate next-to-next-to-next-to-leading order (aN³LO). We construct an approximation to the N³LO splitting functions that includes all available partial information from both fixed-order computations and from small and large x resummation, and estimate the uncertainty on this approximation by varying the set of basis functions used to construct the approximation. We include known N³LO corrections to deep-inelastic scattering structure functions and extend the FONLL general-mass scheme to $\mathcal{O}(\alpha_s^3)$ accuracy. We determine a set of aN³LO PDFs by accounting both for the uncertainty on splitting functions due to the incomplete knowledge of N³LO terms,

arXiv:2402.18635v1 [hep-ph] 28 Feb 2024

N3LO PDFs : what do we need

- Splitting functions

$$\mu^2 \frac{df_i}{d\mu^2} = P_{ij}(\mu^2) \otimes f_j(\mu^2)$$

- VFNS matching conditions

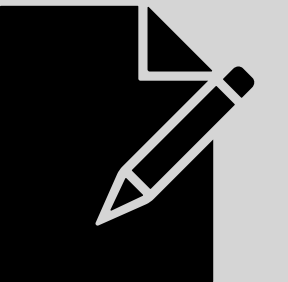
$$f_i^{(n_f+1)}(x, \mu^2) = A_{ij}(x, \alpha_s) \otimes f_j^{(n_f)}(x, \mu^2)$$

- DIS **massless** partonic coefficients

DIS **massive** partonic coefficients

$$F_i(x, Q^2) = \sum_k C_{i,k} \otimes f_k(x, Q^2)$$


- Hadronic coefficients at N3LO



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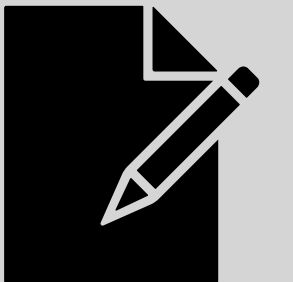
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Full N3LO info not currently available

- Construct approximation for what is missing
- Estimate theory uncertainty

Missing Higher Order Uncertainty (MHOU):
missing higher order terms

Incomplete Higher Order Uncertainty (IHOU):
incomplete higher (N3LO) order terms

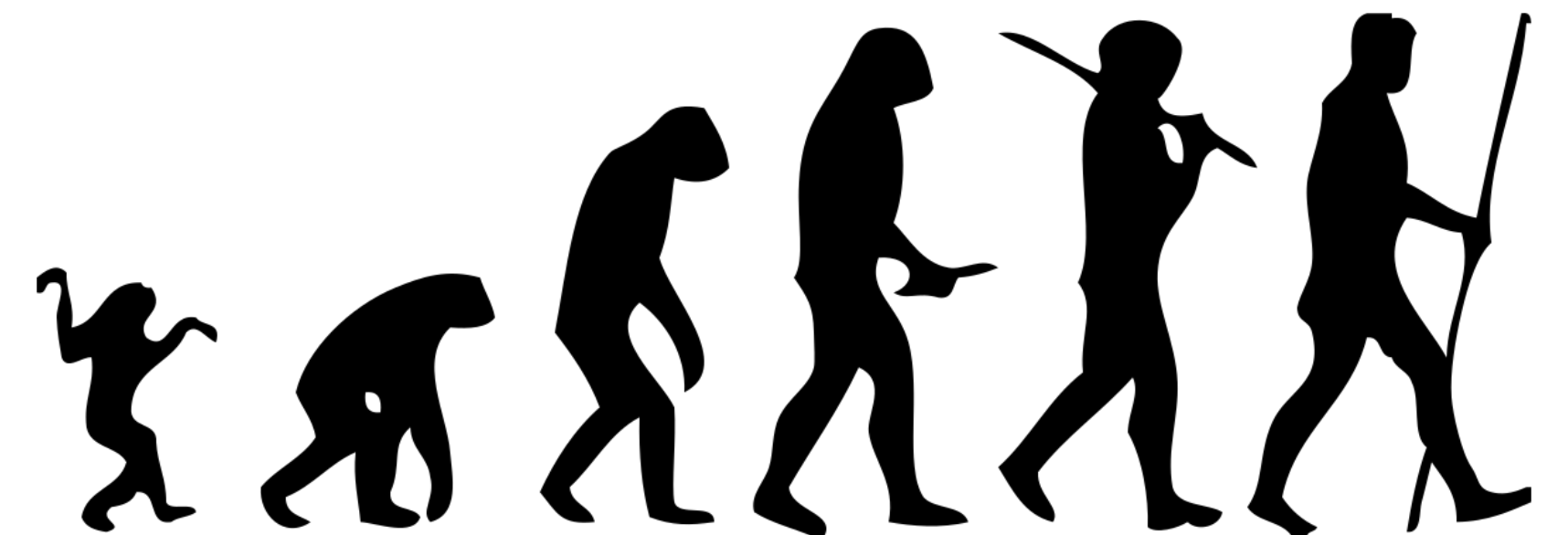
Splitting functions

Nonsinglet

$$\mu^2 \frac{dV}{d\mu^2} = P_{NS,v} \otimes V$$

Singlet

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} g \\ \Sigma \end{pmatrix} = \begin{pmatrix} P_{gg} & P_{gq} \\ P_{qg} & P_{qq} \end{pmatrix} \otimes \begin{pmatrix} g \\ \Sigma \end{pmatrix}$$



Singlet

What we know

$$P_{qq,ps}^{(3)} \quad P_{qg}^{(3)}$$

$$P_{gq}^{(3)} \quad P_{gg}^{(3)}$$

- 5 (10) lowest Mellin moments
[[Phys.Lett.B 825 \(2022\)](#), [Phys.Lett.B 825 \(2022\)](#), [Phys.Lett.B 842 \(2023\)](#), [Phys.Lett.B 849 \(2024\)](#)]
- Small- x limit [[JHEP 06 \(2018\) 145](#)]
- Large- x limit [[JHEP 09 \(2022\) 155](#), [JHEP 04 \(2020\) 018](#), [Nucl.Phys.B 832 \(2010\)](#)]
- large- n_f limit, i.e. $\mathcal{O}(n_f^2)$, $\mathcal{O}(n_f^3)$
[[Nucl. Phys. B 915 \(2017\) 335–362](#), [Phys. Lett. B 848 \(2024\)](#), [JHEP 01 \(2024\) 029](#)]

Our approximation

$$\gamma_{ij}^{(3)}(N) = \underbrace{\gamma_{ij,n_f}^{(3)}(N)}_{\text{large-}n_f \text{ limit}} + \underbrace{\gamma_{ij,N \rightarrow \infty}^{(3)}(N)}_{\text{large-}x \text{ limit}} + \underbrace{\gamma_{ij,N \rightarrow 0}^{(3)}(N)}_{\text{small-}x \text{ limit}} + \underbrace{\gamma_{ij,N \rightarrow 1}^{(3)}(N)}_{\text{remainder}} + \tilde{\gamma}_{ij}^{(3)}(N)$$

large- n_f limit

large- x limit

small- x limit

remainder

- Expanded on a set of basis function
- Coefficients of the expansion determined by known moments
- Reproduce leading and subleading N-space poles with unknown coefficients

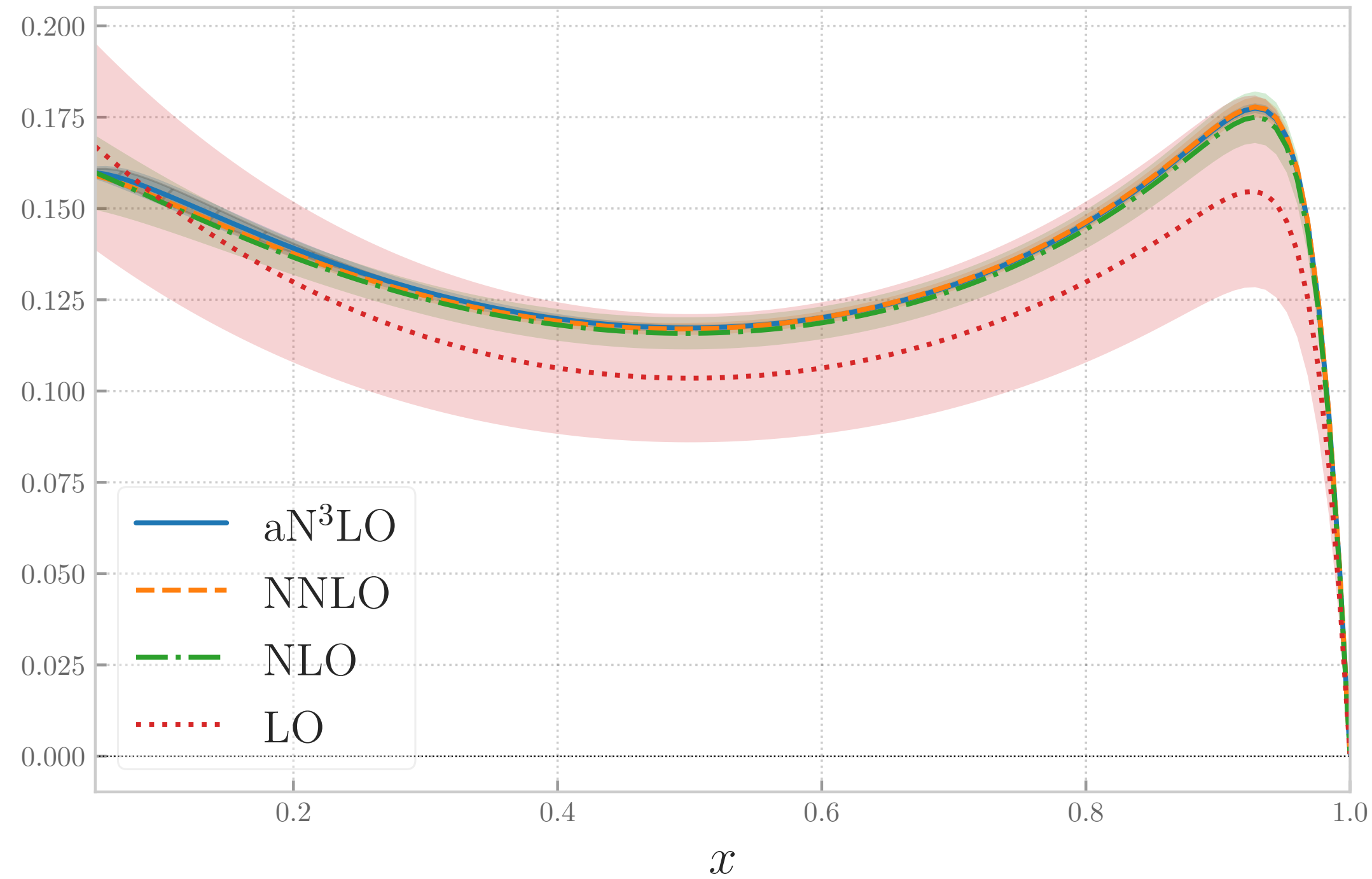
Large-N and small-N most leading contributions

Further subleading terms

$\gamma_{gg}^{(3)}(N)$	$G_1^{gg}(N)$	$\mathcal{M}[(1-x)\ln^3(1-x)](N)$
	$G_2^{gg}(N)$	$\frac{1}{(N-1)^2}$
	$G_3^{gg}(N)$	$\frac{1}{N-1}$
	$\{H_1^{gg}(N), H_2^{gg}(N)\}$	$\frac{1}{N^4}, \frac{1}{N^3}, \frac{1}{N^2}, \frac{1}{N+1}, \frac{1}{N+2}, \mathcal{M}[(1-x)\ln^2(1-x)](N), \mathcal{M}[(1-x)\ln(1-x)](N)$

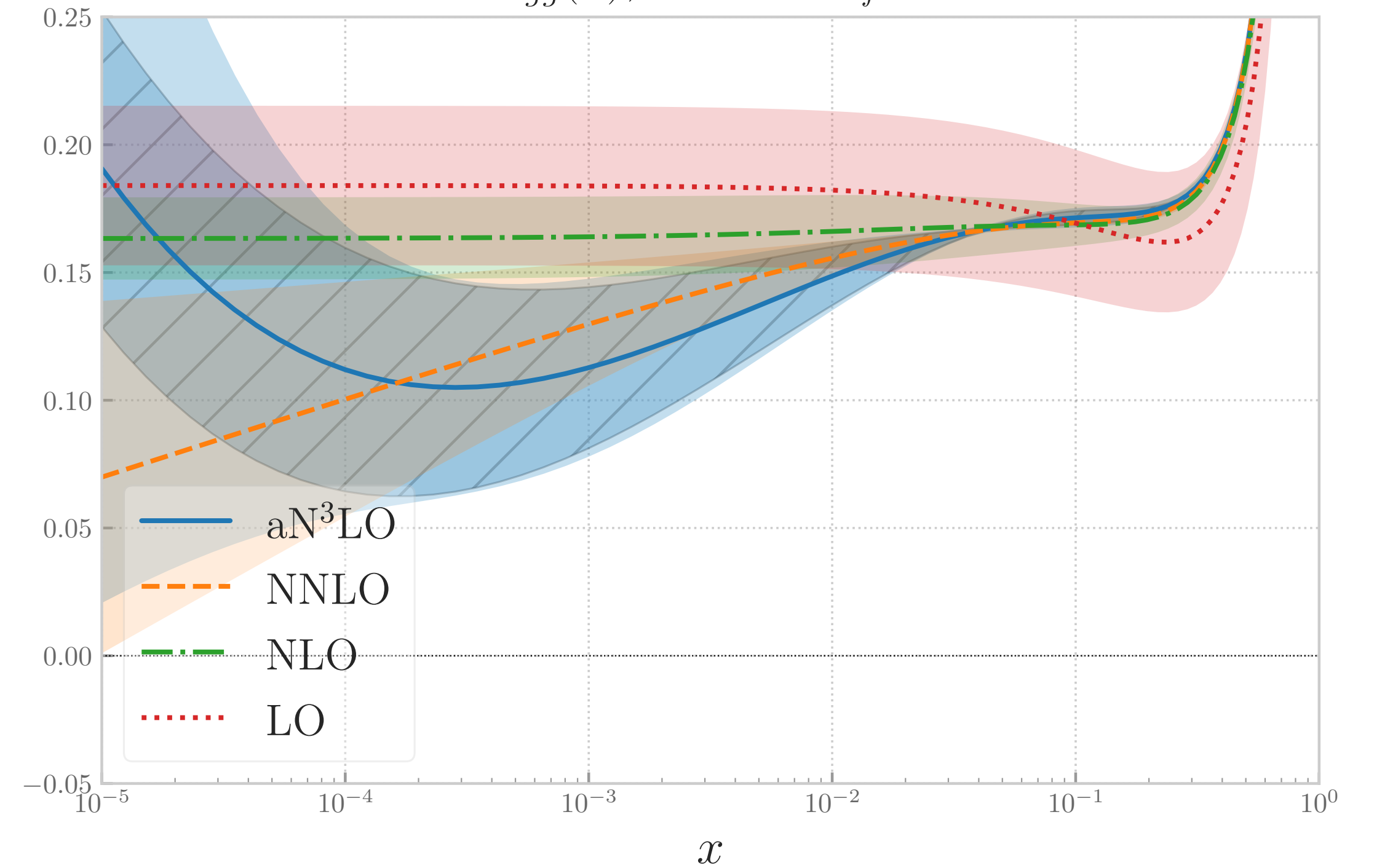
P_{gg}

Large-x

 $x(1-x)P_{gg}(x), \alpha_s = 0.2, n_f = 4$ 

- Good perturbative convergence
- IHOU and MHOU are both negligible

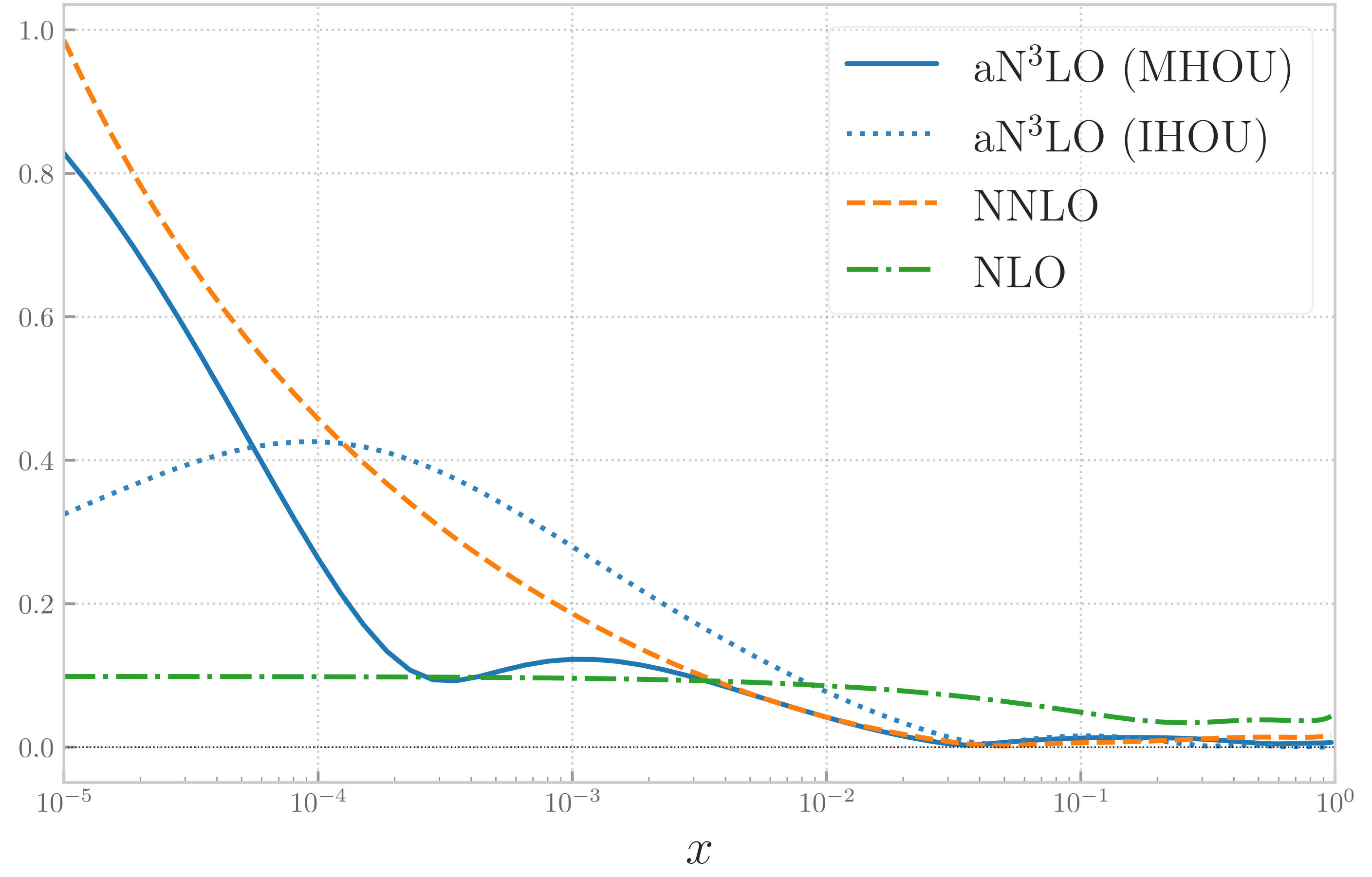
small-x

 $xP_{gg}(x), \alpha_s = 0.2, n_f = 4$ 

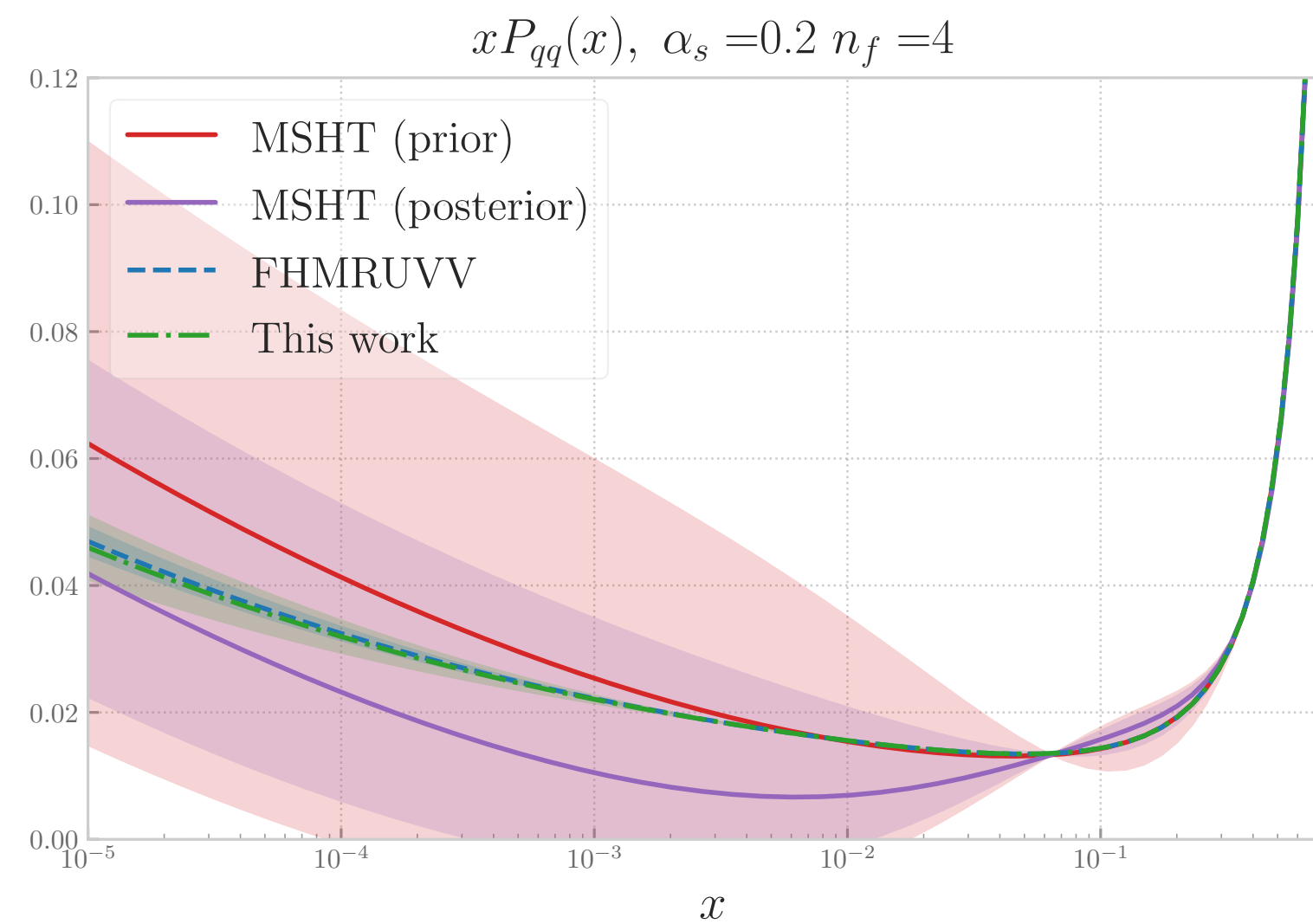
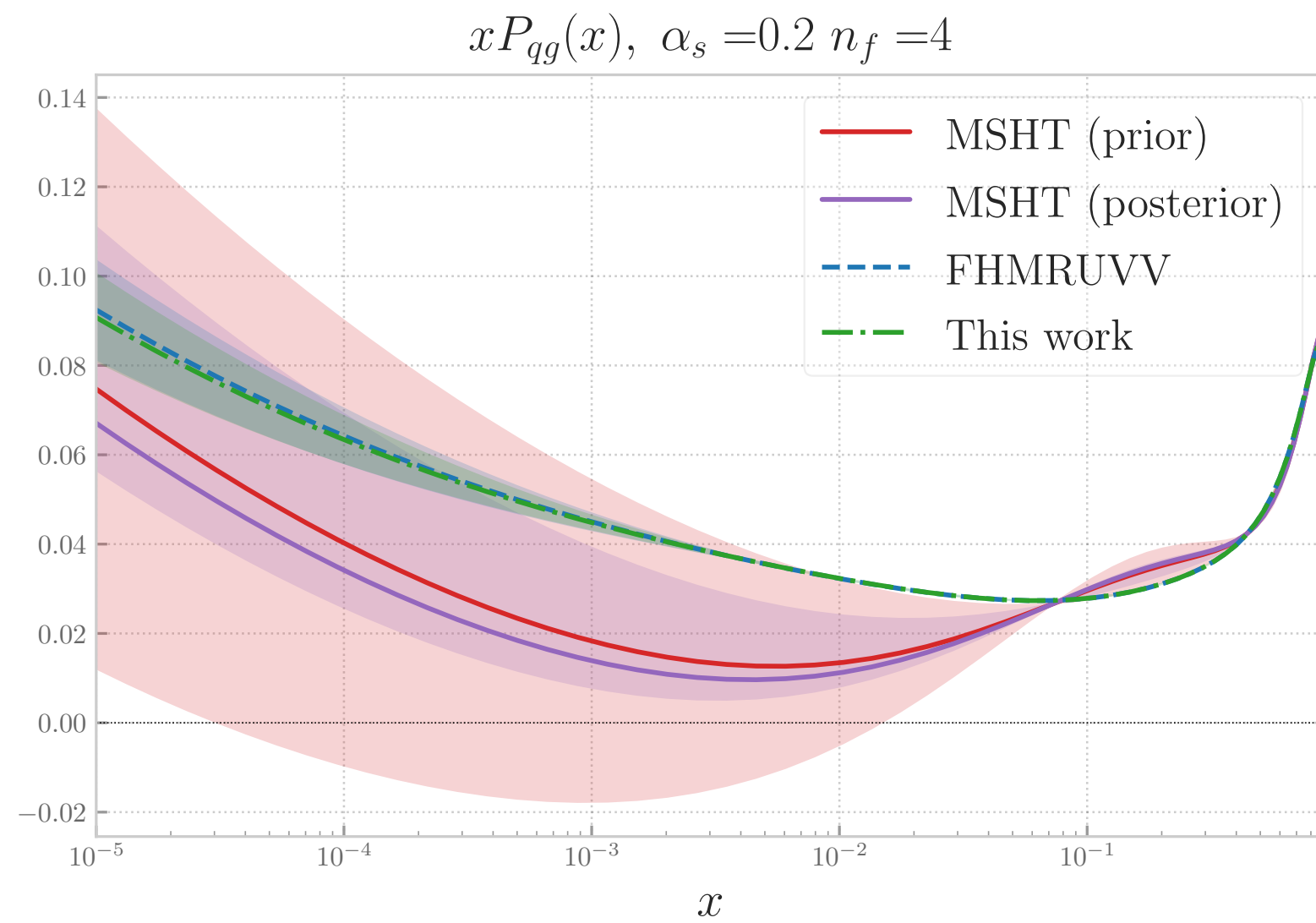
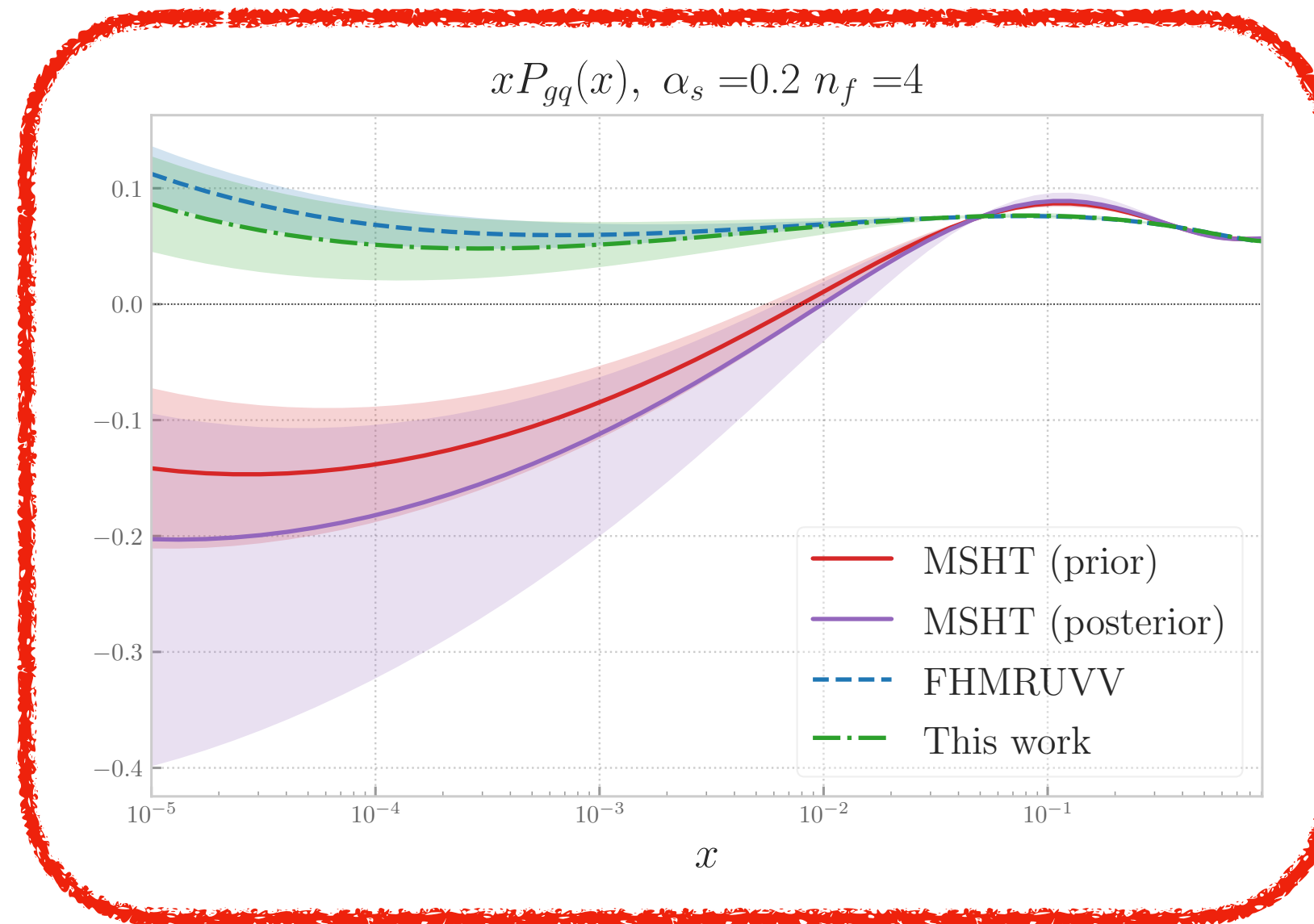
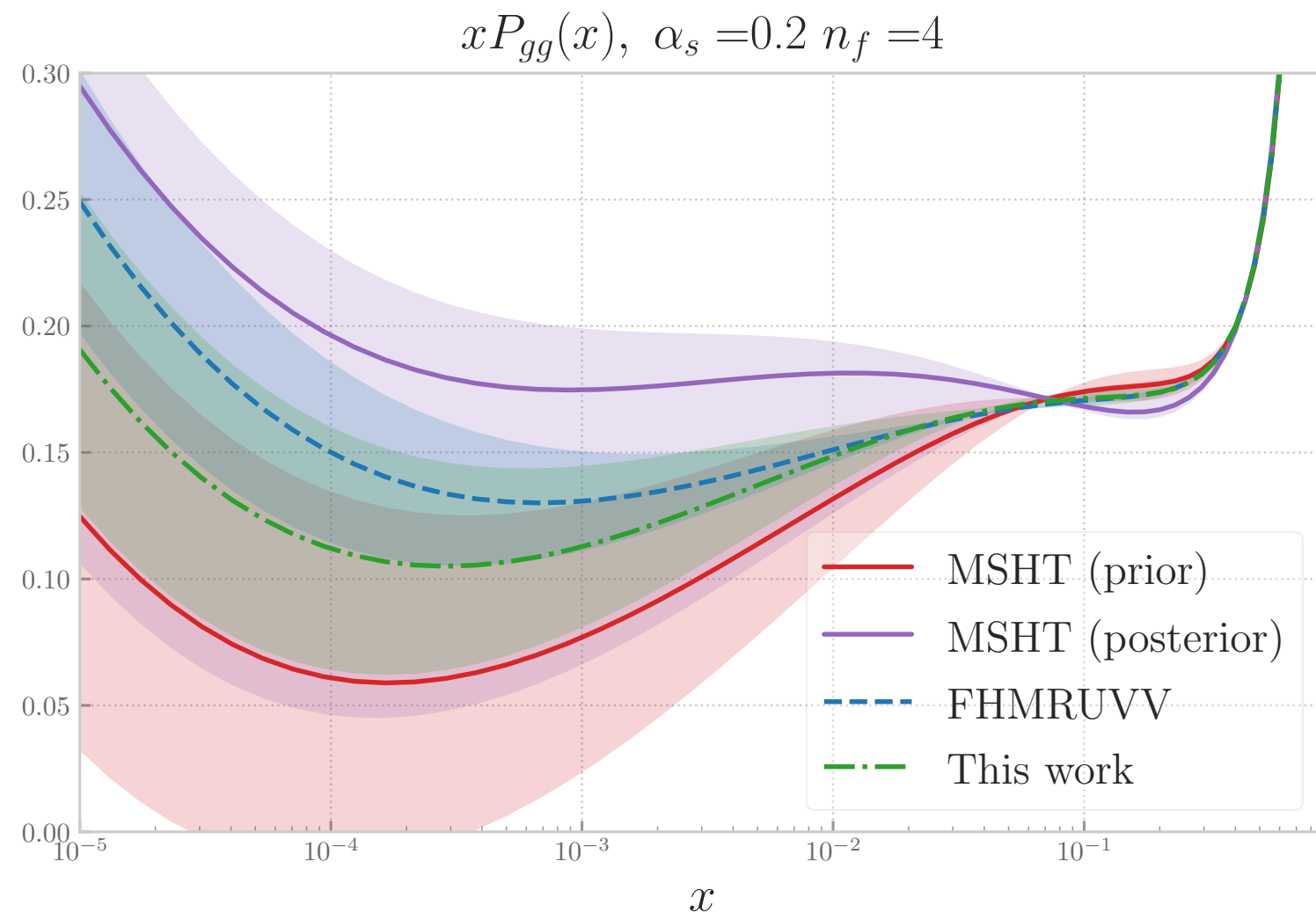
- aN³LO and NNLO result agree within uncertainties
- aN³LO uncertainties are sizeable
- IHOU dominate the gluon sector

singlet: small-x

$$\sigma_{gg}/P_{gg}(x), \alpha_s = 0.2 \ n_f = 4$$



Comparison to MSHT aN3LO splitting function



- MSHT: to estimate IHOU additional nuisance parameters which are fitted to the data
- Nuisance params are taken as a prior and by looking at the data they determine posterior
- Biggest difference for P_{qg} where MSHT is very different from both NNPDF and previous approximation FHMRUVV [[JHEP 10 \(2017\) 041](#)]

Partonic cross-sections



DIS

- DIS structure functions F_2, F_L, F_3 known at N3LO in the massless limit
DIS NC [[arxiv:9605317](#), [arxiv:0411112](#), [arxiv:0504242](#)]
DIS CC [[arxiv:0812.4168](#), [arxiv:1606.08907](#)]
- DIS massive structure functions can be approximated from known limits and interpolation functions [[arXiv:2401.12139](#), [Barontini, Bonvini, Laurenti, in preparation](#)]

$$C_{i,k}^{(3)}(x, m_h^2/Q^2) = C_{i,k}^{(3),thr} f_1(x) + C_{i,k}^{(3),asy} f_2(x)$$

Threshold limit

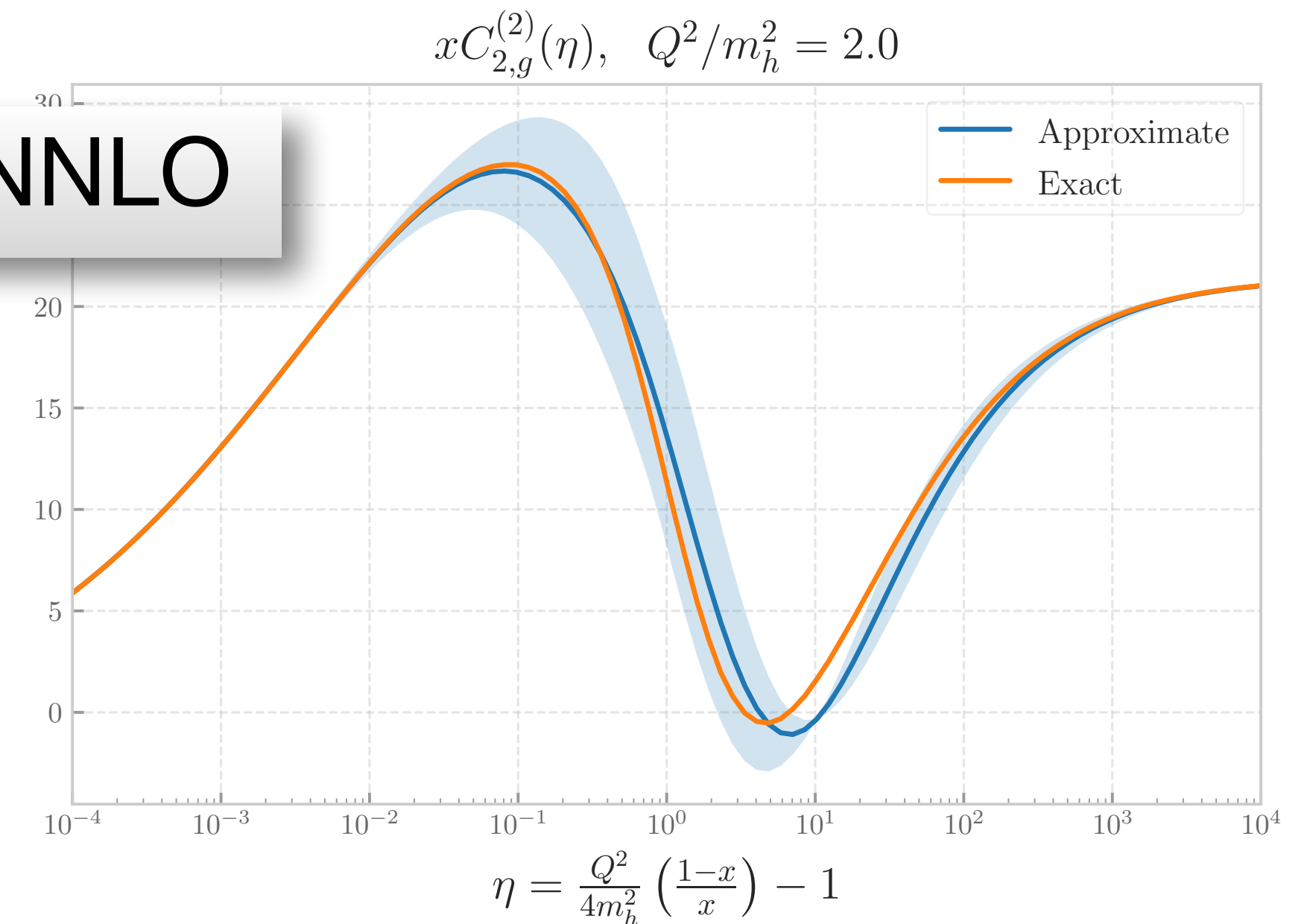
$$x \rightarrow x_{max} = \frac{Q^2}{4m_h^2 + Q^2}$$

High energy limit $x \rightarrow 0$

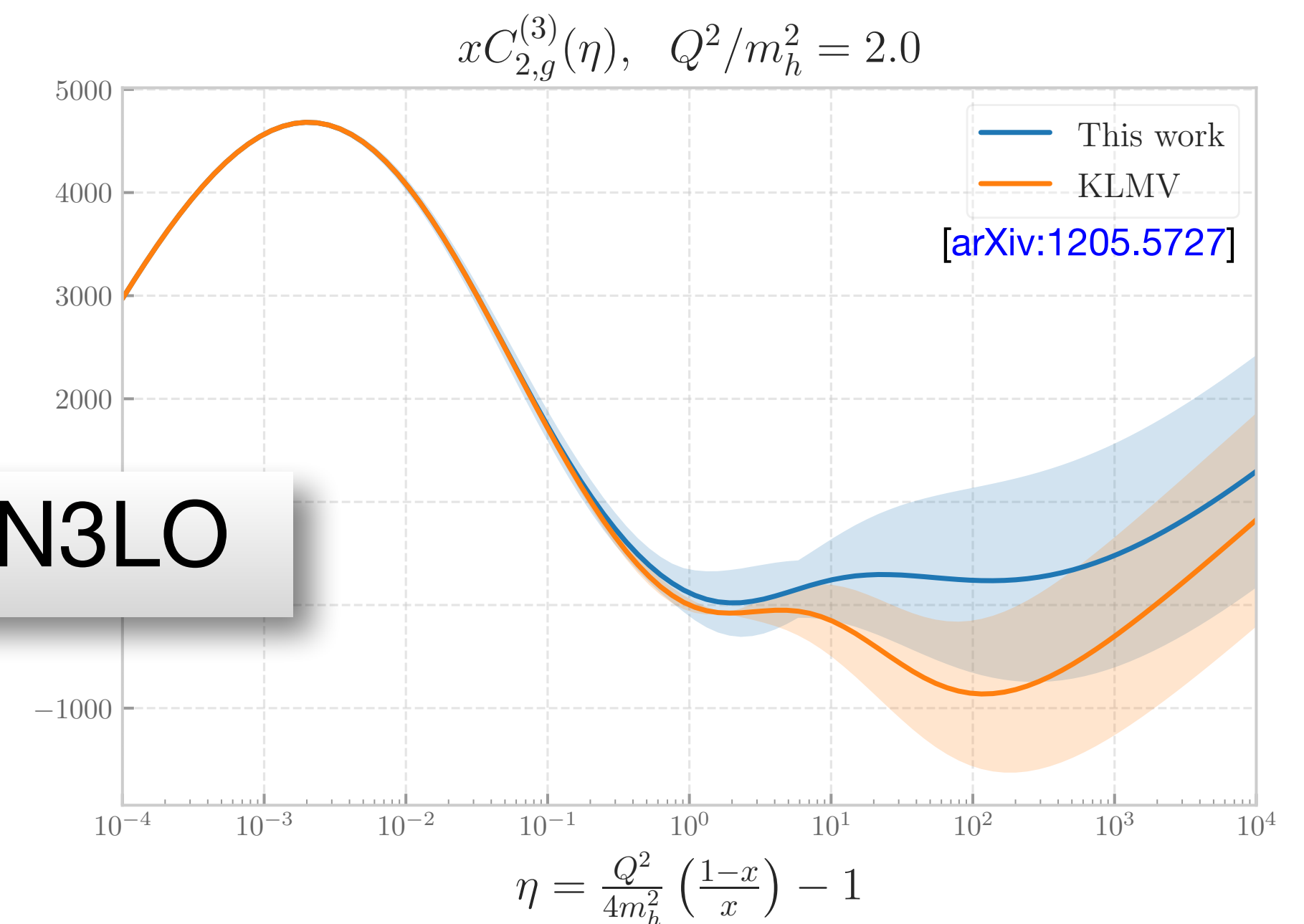
Asymptotic limit $Q^2 \gg m_h^2$

Uncertainty of the approximate coefficient function is built by varying interpolating functions

NNLO



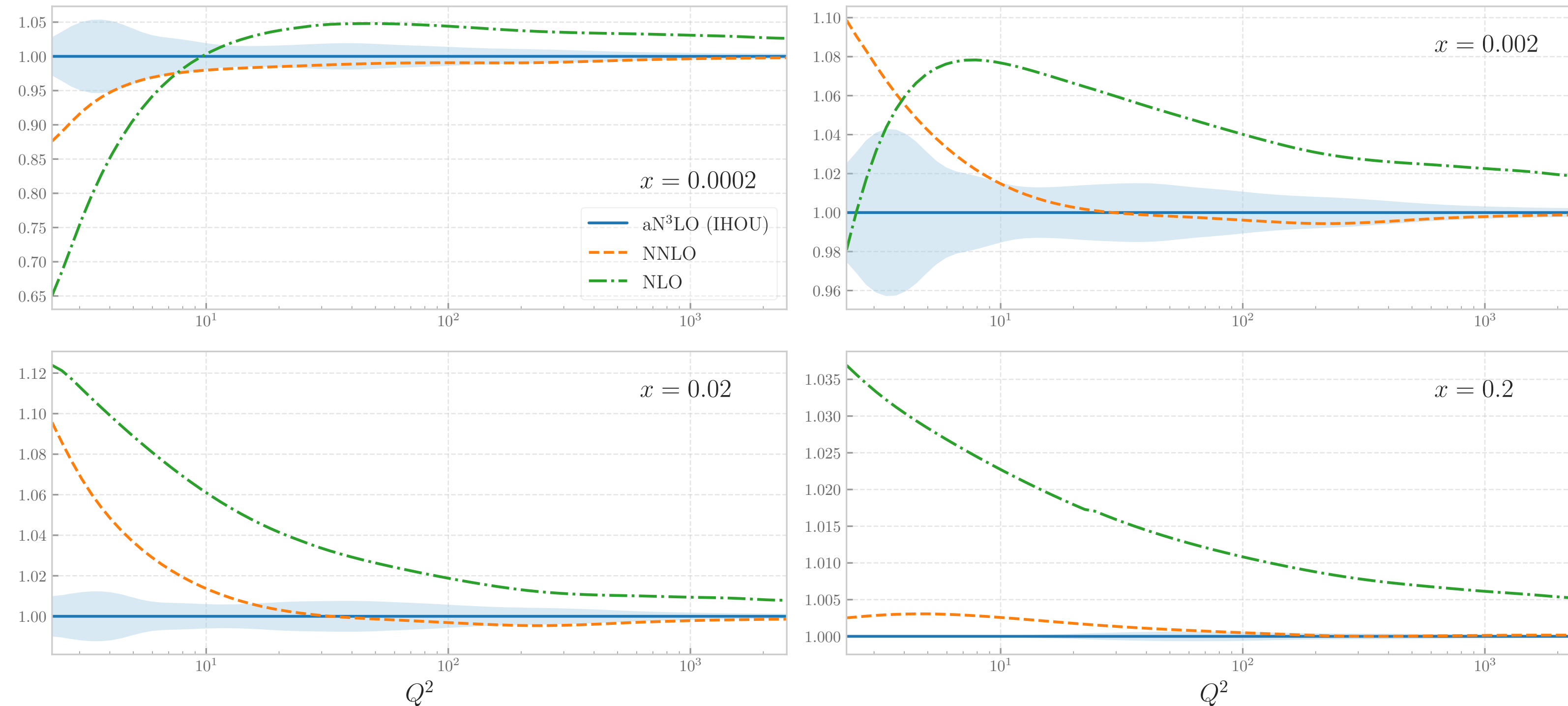
N3LO



Extension to FONLL accounting for both massive and massless contributions at $\mathcal{O}(\alpha_s^3)$ (FONLL-E)

$$F_i^{FONLL} = F_i^{(n)}(x, Q^2, m_h^2) + F_i^{(n+1)}(x, Q^2) - F_i^{(n,0)}(x, \ln(Q^2/m_h^2))$$

$F_2^{(\text{tot})}(x, Q^2)$, ratio to aN³LO






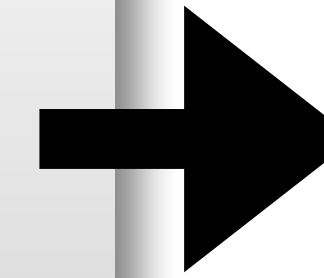
- aN3LO corrections to NNLO lower than 2% for $Q^2 > 10 \text{ GeV}^2$ and at most 10% around charm mass
- aN3LO corrections larger than IHO in a significant region

Hadronic processes

N3LO corrections are not included

Hadronic processes

- N3LO corrections pub available for inclusive NC and CC DY (differential distribution at the level of leptonic observables also computed but not available) 
- No N3LO calculations available for other data of NNPDF4.0 
- Rapidity dependence of K factors 



No inclusion of hadronic data in default set.

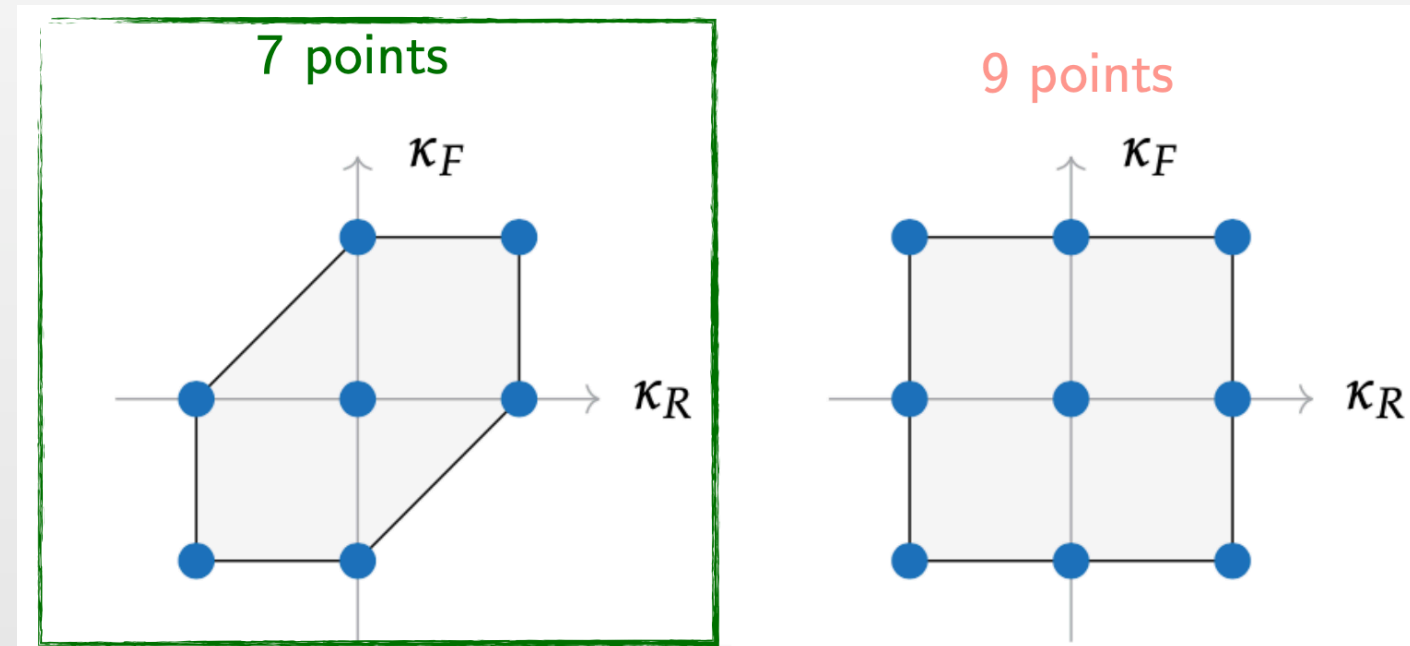
dedicated variant to assess the impact of available data/computations (backup)

Dataset	Ref.	n_{dat}	Kin ₁	Kin ₂ [GeV]	C -factor N ³ LO/NNLO
ATLAS high-mass DY 7 TeV	[103]	13	$ \eta_e \leq 2.1$	$116 \leq m_{\ell\ell} \leq 1500$	$d\sigma/dm_{\ell\ell}$
ATLAS Z 7 TeV ($\mathcal{L} = 35 \text{ pb}^{-1}$)	[104]	8	$ \eta_e, y_Z \leq 3.2$	$Q = m_Z$	$d\sigma/dm_{\ell\ell}$ ($66 < m_{\ell\ell} < 150$)
ATLAS Z 7 TeV ($\mathcal{L} = 4.6 \text{ fb}^{-1}$) CC	[105]	24	$ \eta_e, y_Z \leq 2.5, 3.6$	$Q = m_Z$	$d\sigma/dm_{\ell\ell}$ ($46 < m_{\ell\ell} < 116$)
ATLAS $\sigma_{W,Z}^{\text{tot}}$ 13 TeV	[106]	3	—	$Q = m_W, m_Z$	σ

MHOU

Uncertainties due to perturbative truncation of partonic cross sections and anomalous dimensions

[[arXiv:1905.04311](https://arxiv.org/abs/1905.04311), [arXiv:1906.10698](https://arxiv.org/abs/1906.10698), [arXiv:2401.10319](https://arxiv.org/abs/2401.10319)]



Process categories
DIS NC
DIS CC
DY NC
DY CC
Top pair
Single top
Single inclusive jets
Prompt photon
Dijets

- **Splitting functions (N4LO)**
- **DIS partonic coefficients (N4LO)**
- **Hadronic partonic xsec (N3LO)**

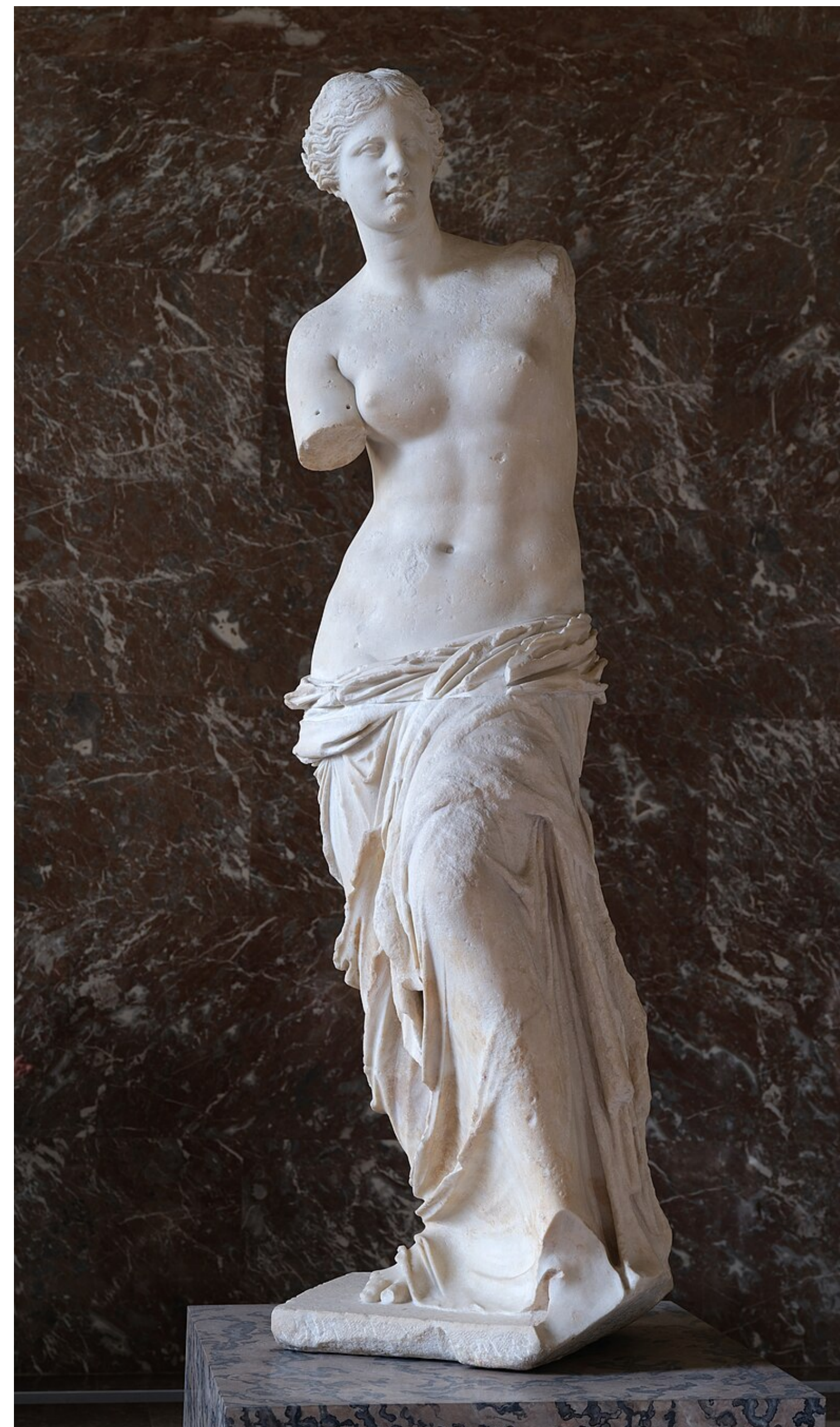
IHOU

Uncertainties due to incomplete knowledge of N3LO corrections

- **Splitting functions**
- **DIS partonic coefficients**

$$\text{COV}^{tot} = \text{COV}^{exp} + \text{COV}^{MHOU} + \text{COV}^{IHOU}$$

aN3LO PDFs



aN3LO default sets

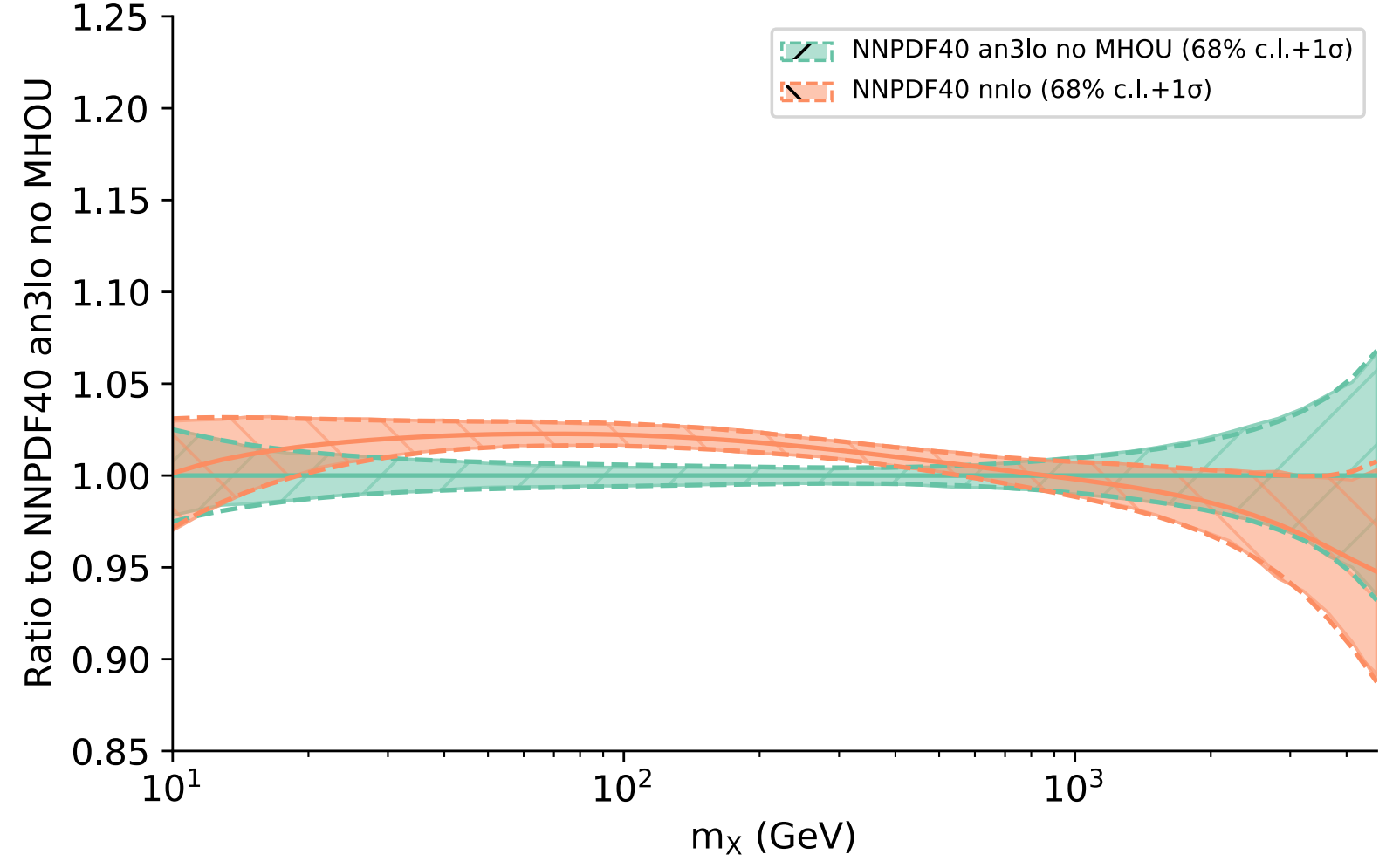
	aN3LO splitting functions	N3LO massless DIS + aN3LO massive	N3LO corrections on hadronic data	MHOU on hadronic data	MHOU on DIS data	MHOU on anomalous dimension
NNPDF40 aN3LO	✓	✓	✗	✓	✗	✗
NNPDF40 aN3LO MHOU	✓	✓	✗	✓	✓	✓

- Same dataset and methodology used for NNPDF4.0
- Results provided both with and without MHOU
- Impact of the available hadronic data is assessed in variant fits (backup slides)

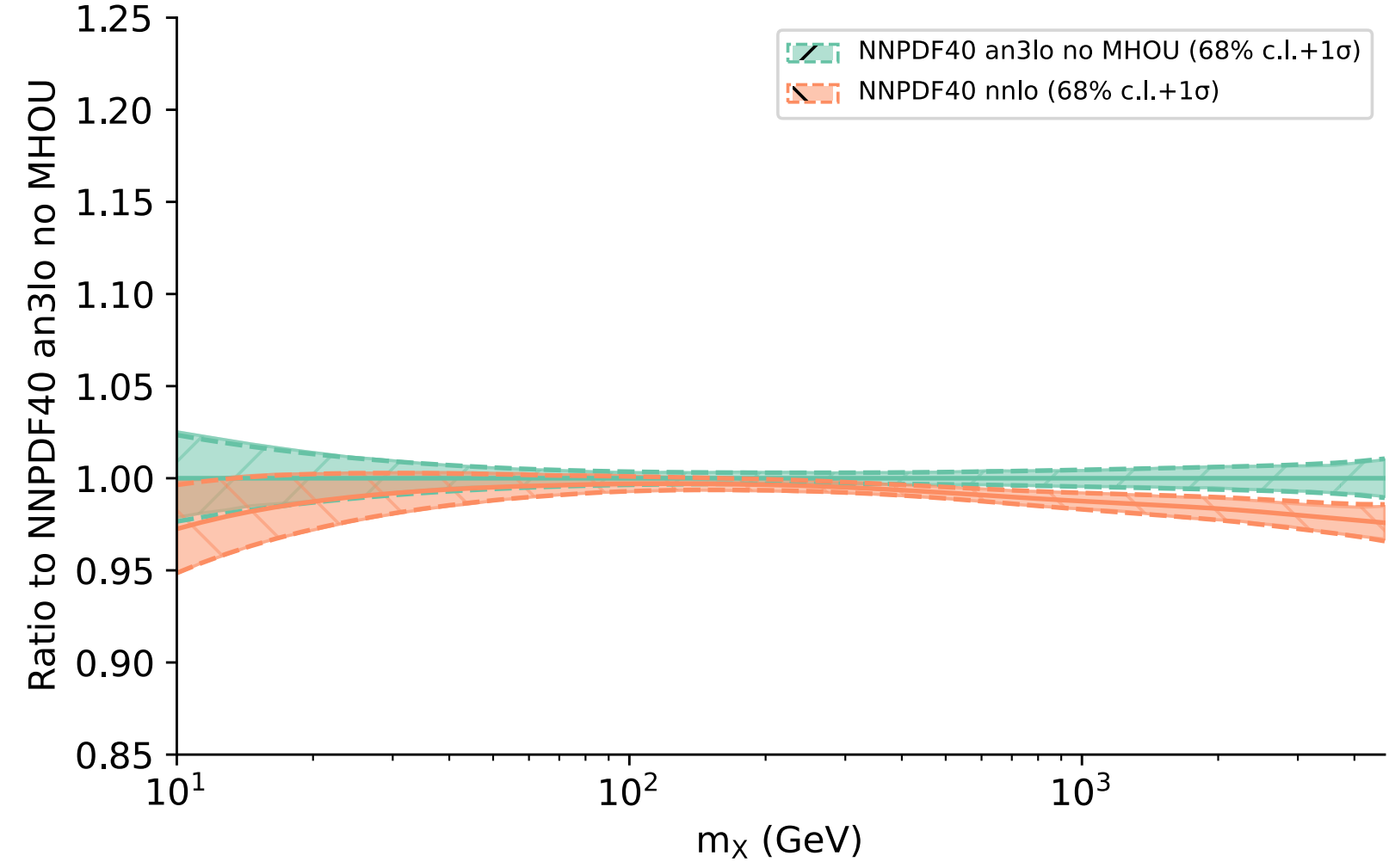
aN3LO partonic luminosity (NNPDF)

$$\mathcal{L}_{ij}(m_x, \sqrt{s}) = \frac{1}{s} \int_{\tau}^1 \frac{dx}{x} f_i(x, m_x) f_j\left(\frac{\tau}{x}, m_x\right)$$

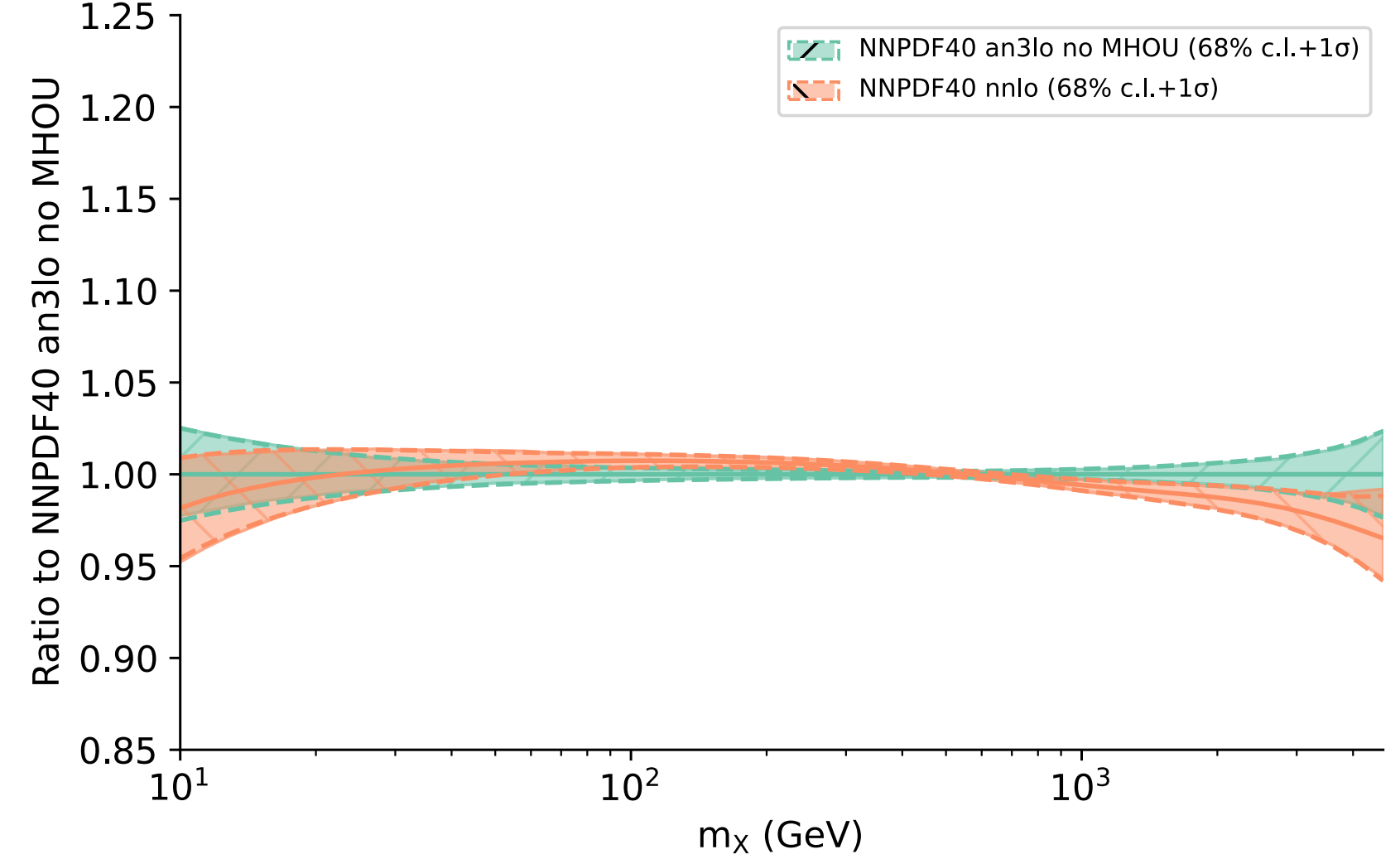
gg luminosity
 $\sqrt{s} = 14$ TeV



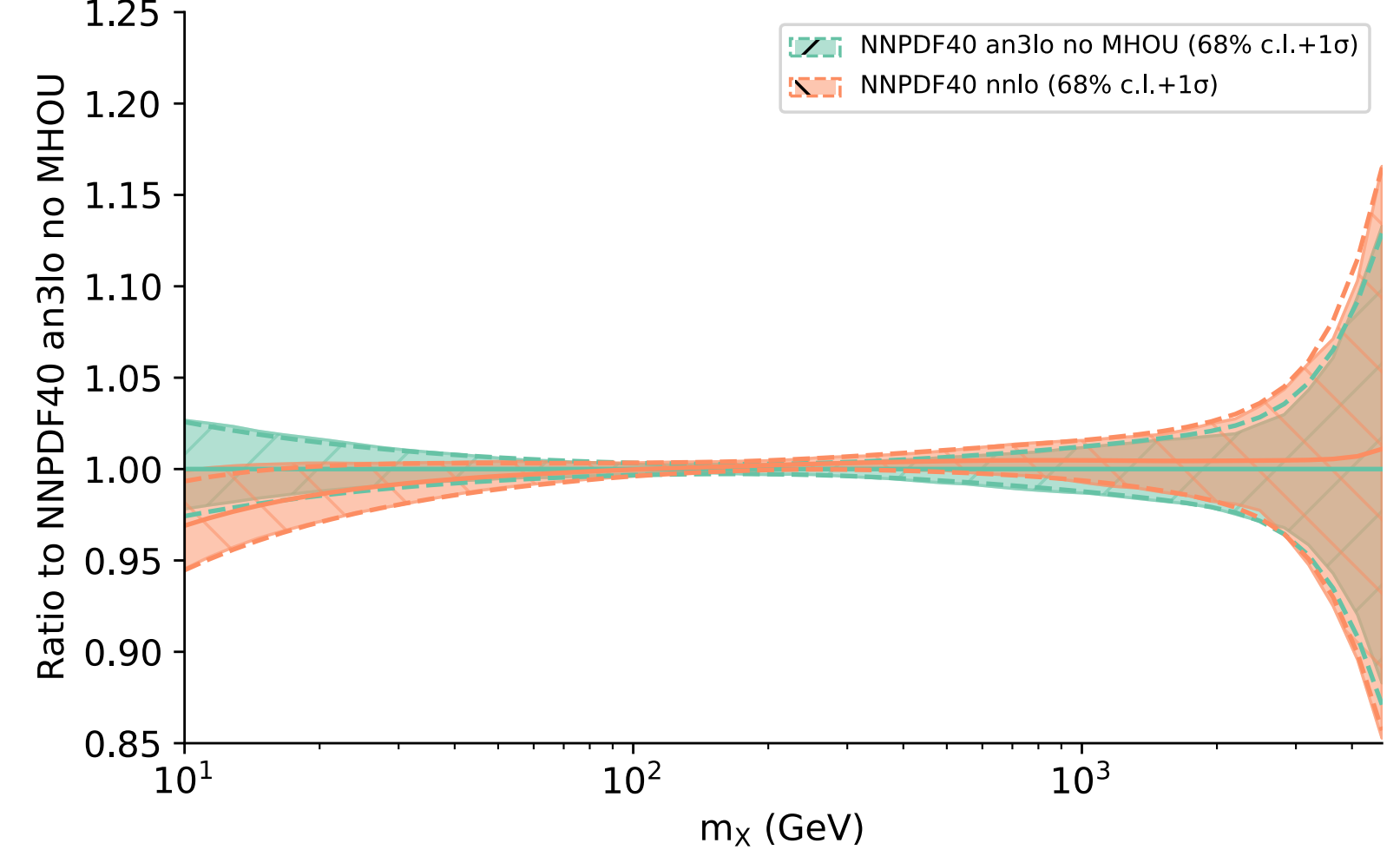
qq luminosity
 $\sqrt{s} = 14$ TeV



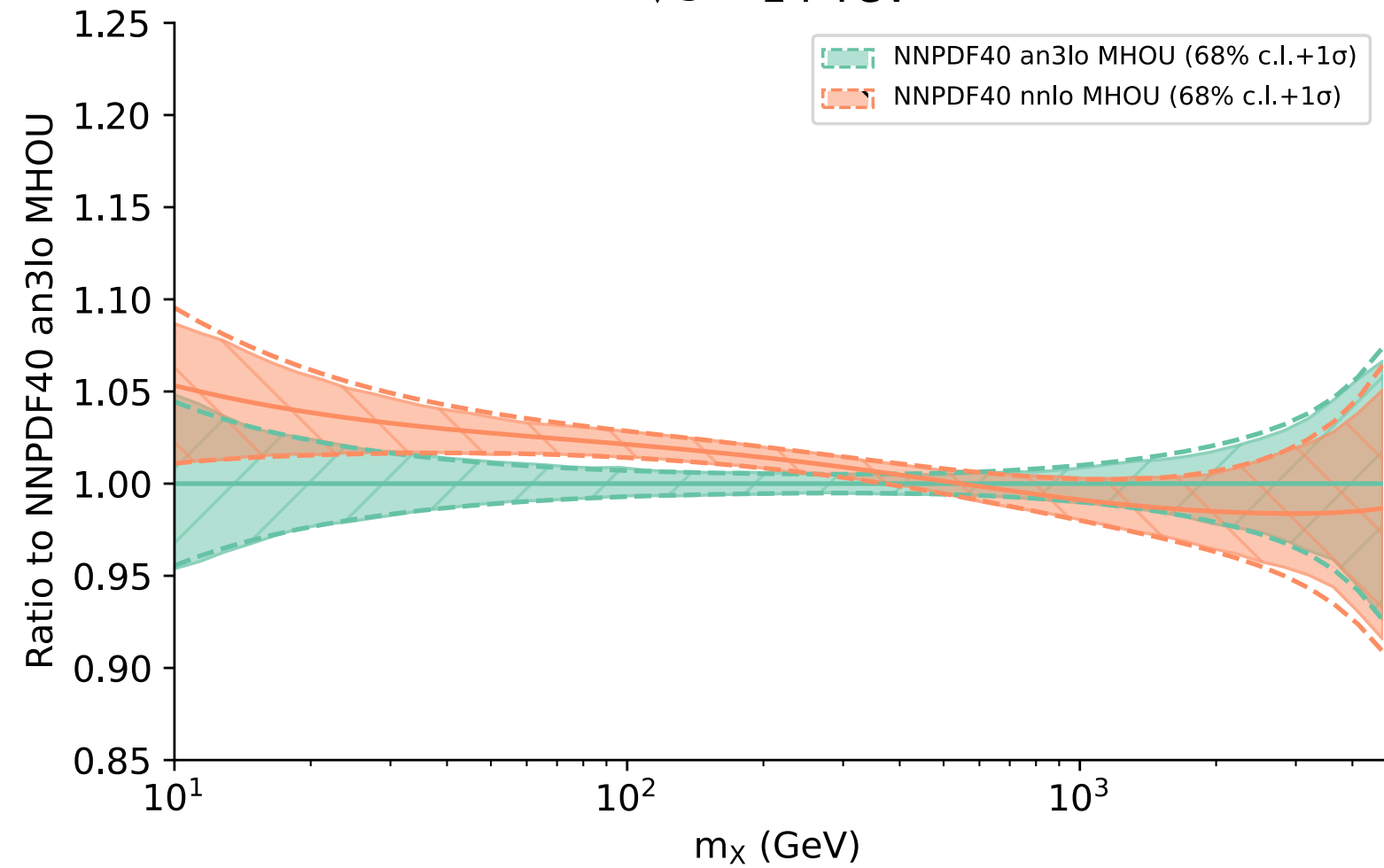
gq luminosity
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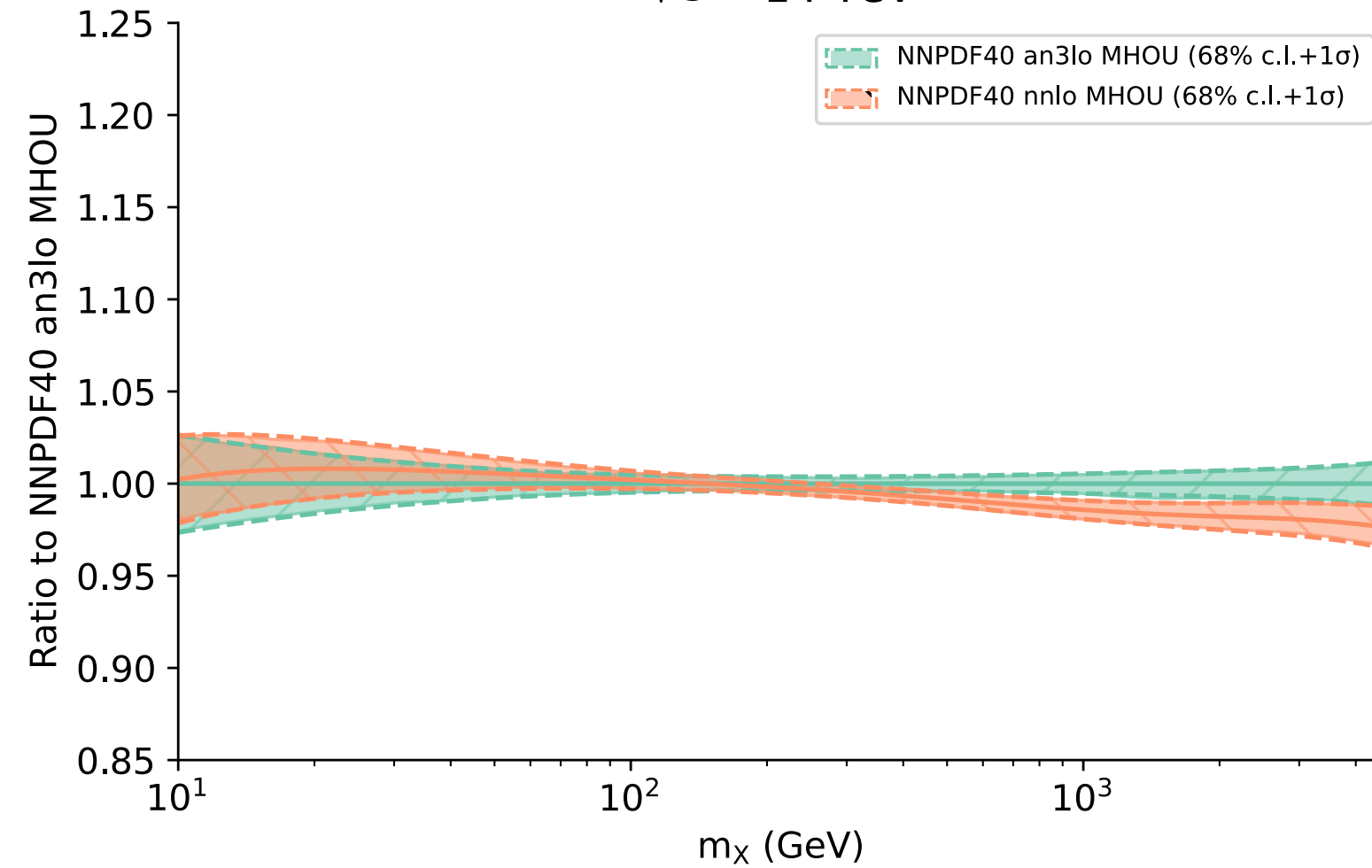
q \bar{q} luminosity
 $\sqrt{s} = 14$ TeV



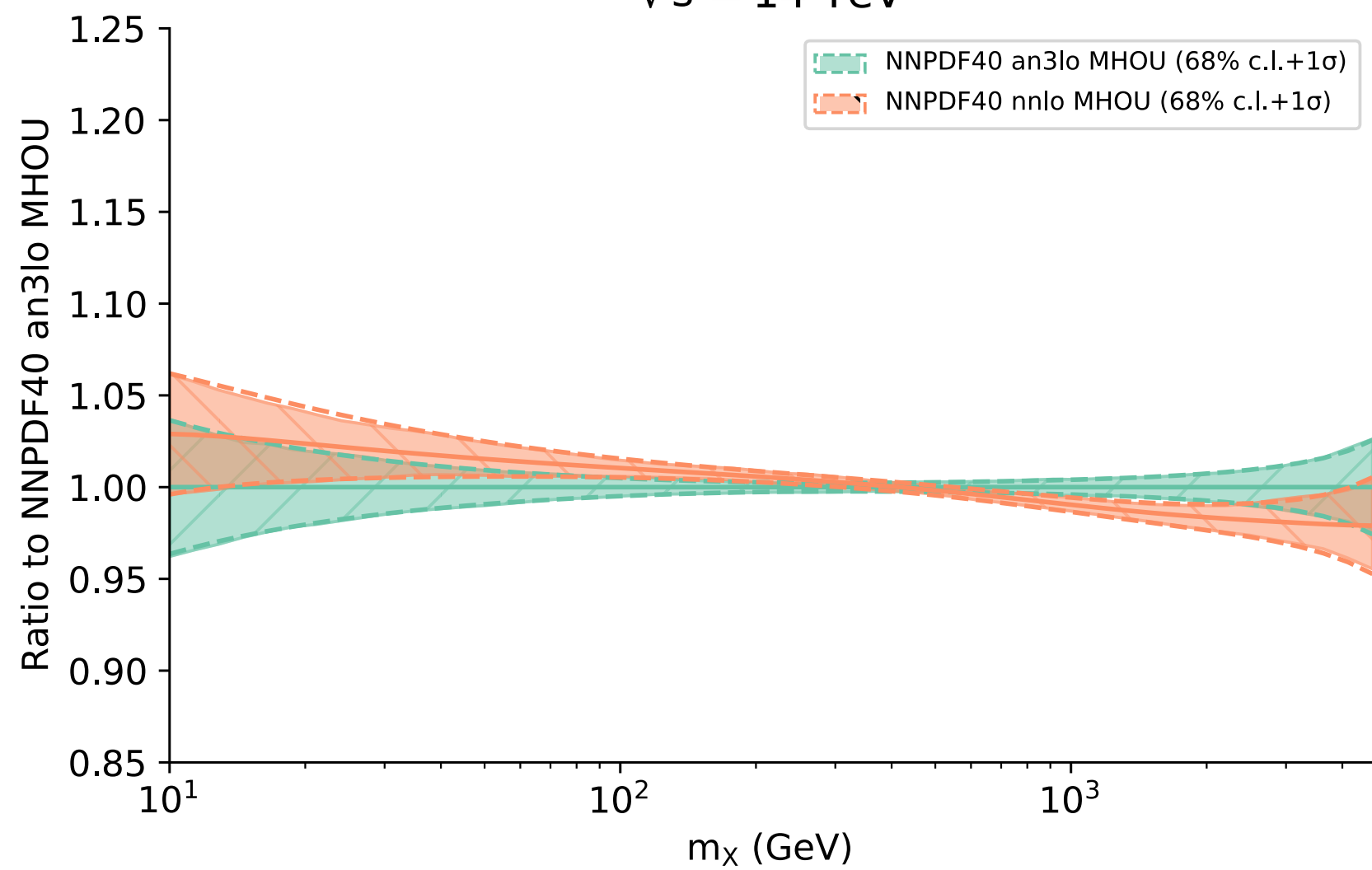
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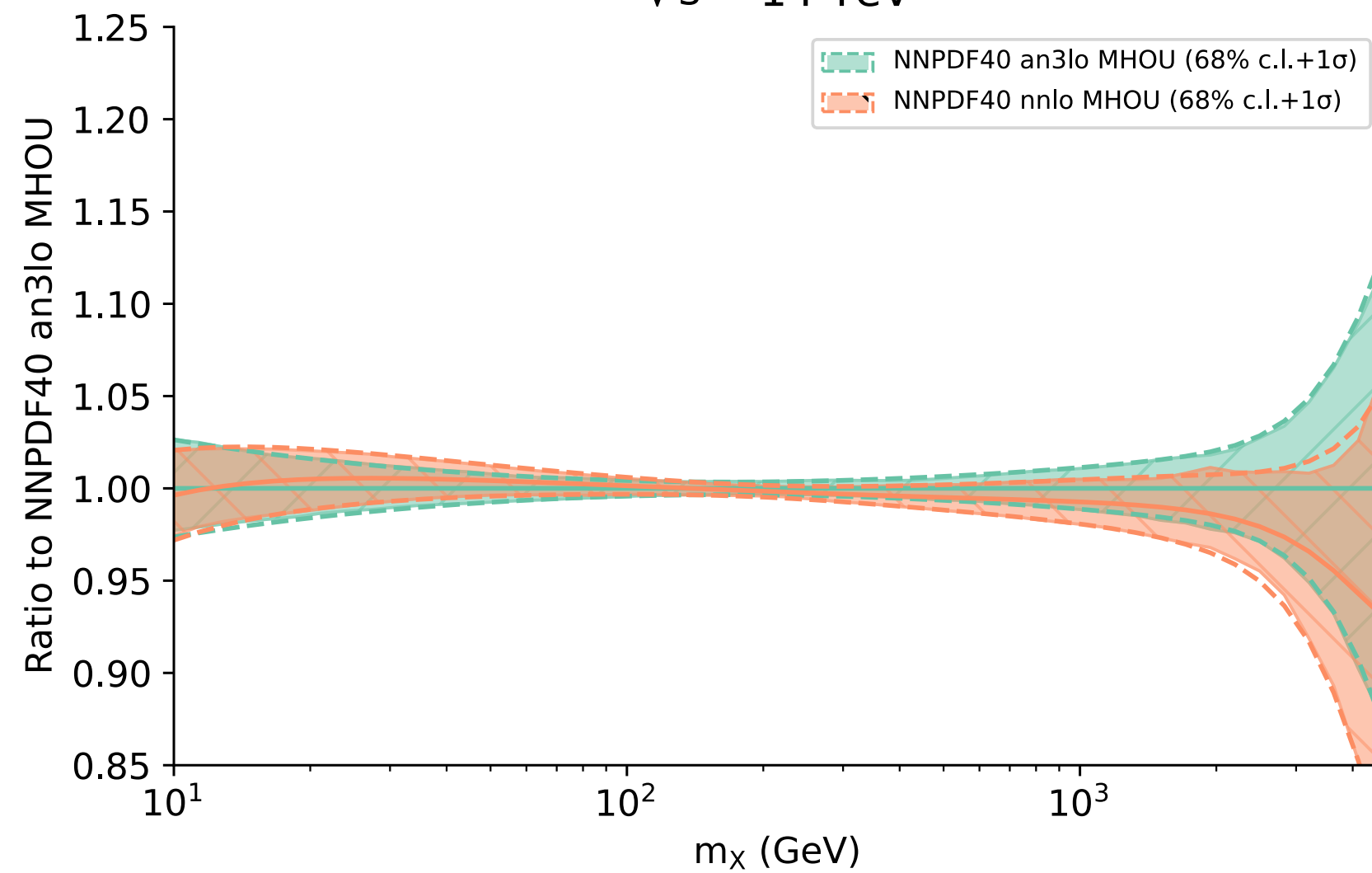
qq luminosity
 $\sqrt{s} = 14$ TeV



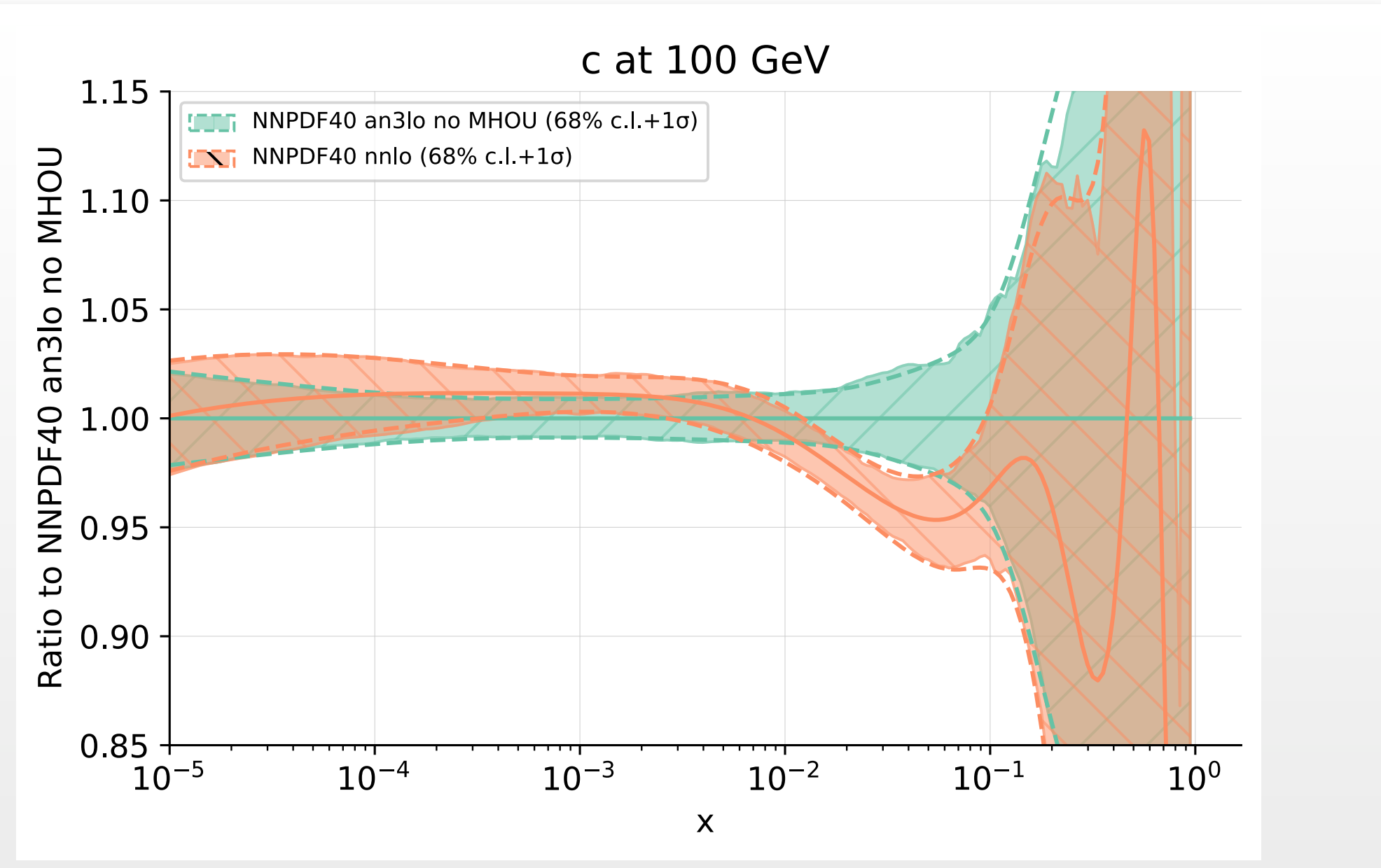
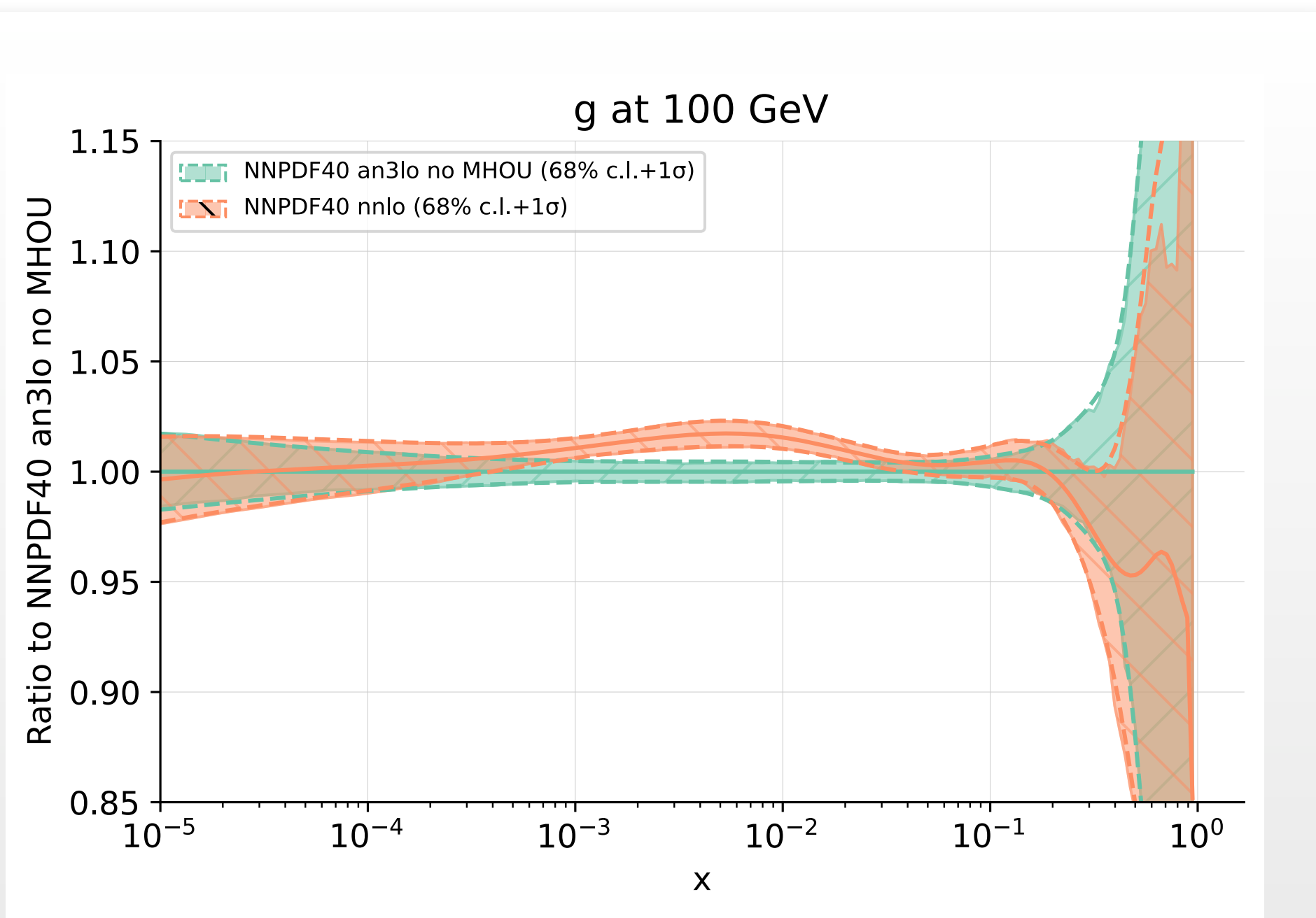
gg luminosity
 $\sqrt{s} = 14$ TeV



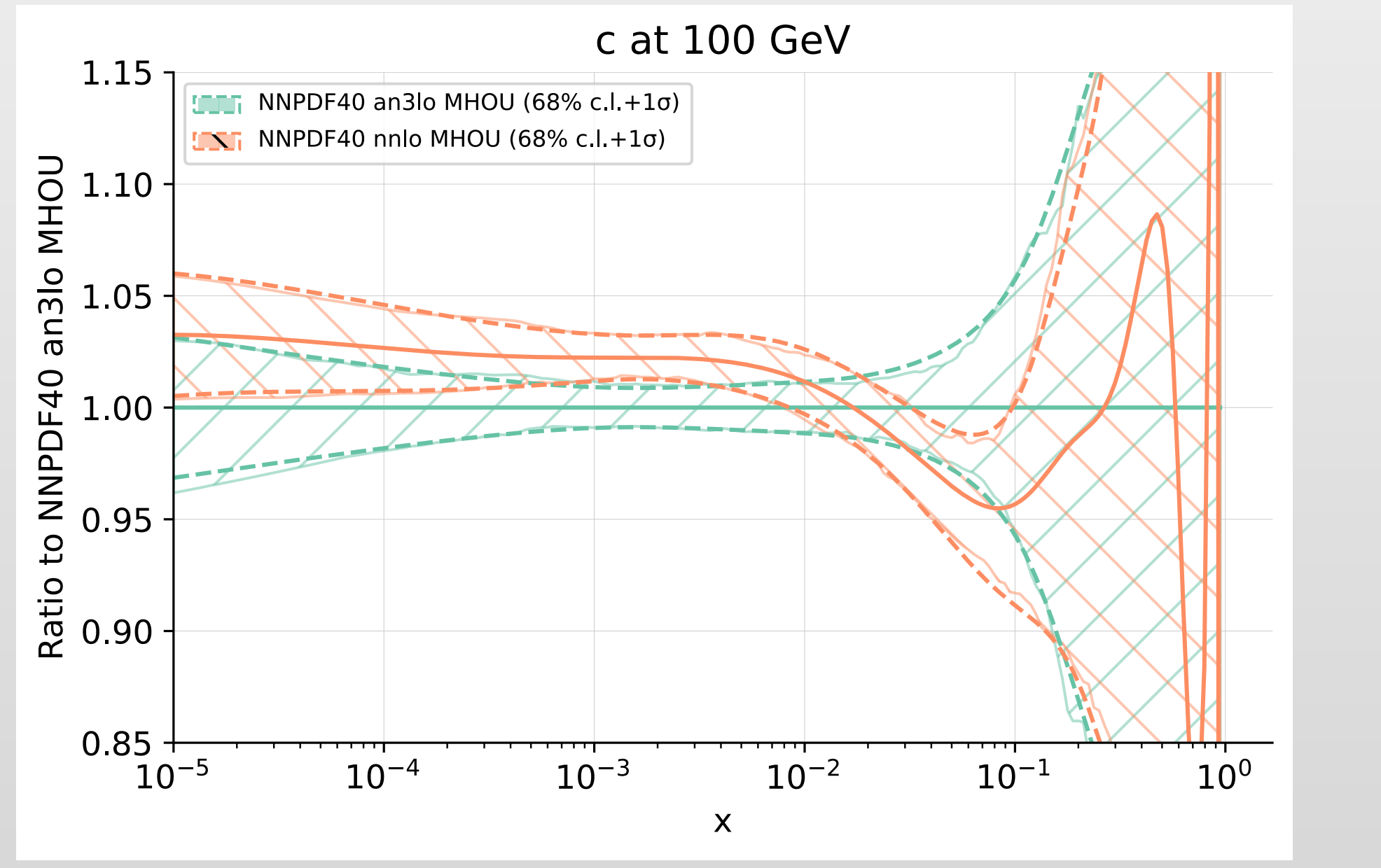
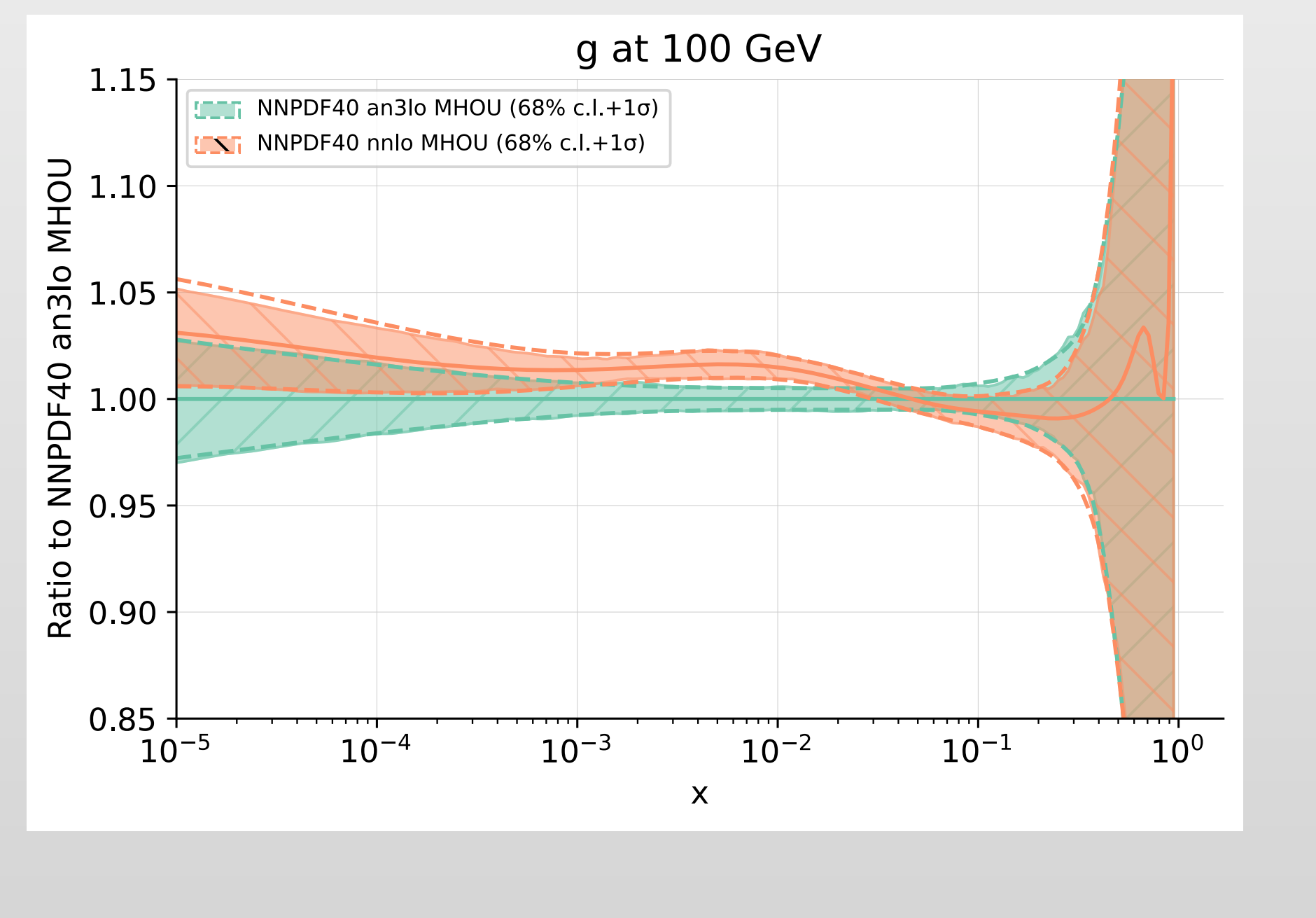
q \bar{q} luminosity
 $\sqrt{s} = 14$ TeV



- Suppression of gg luminosity
- Enhancement of qq luminosity
- Inclusion of MHO improve agreement (increase in uncertainties of $\sim 1\% - 2\%$)

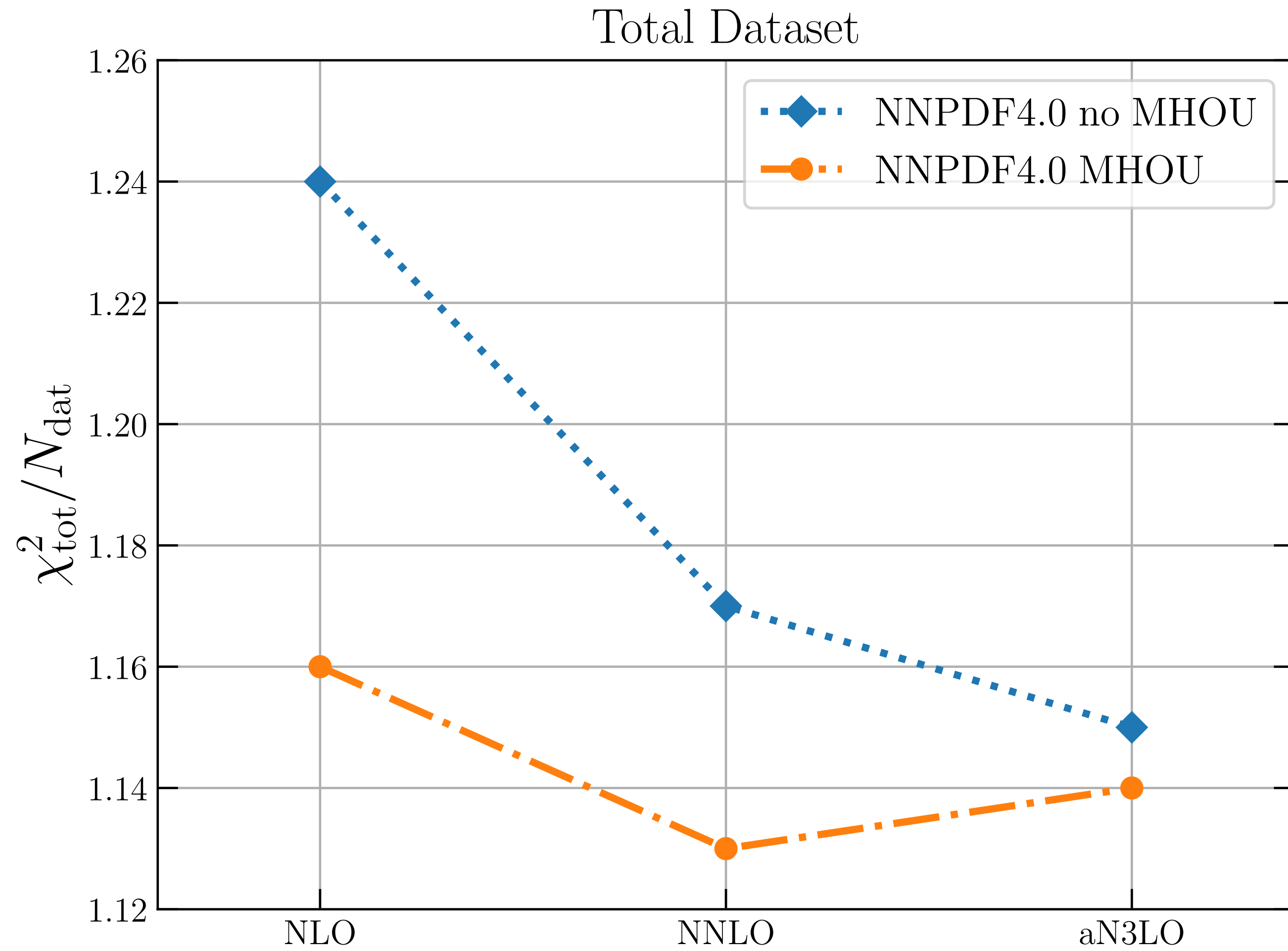


no MHOU



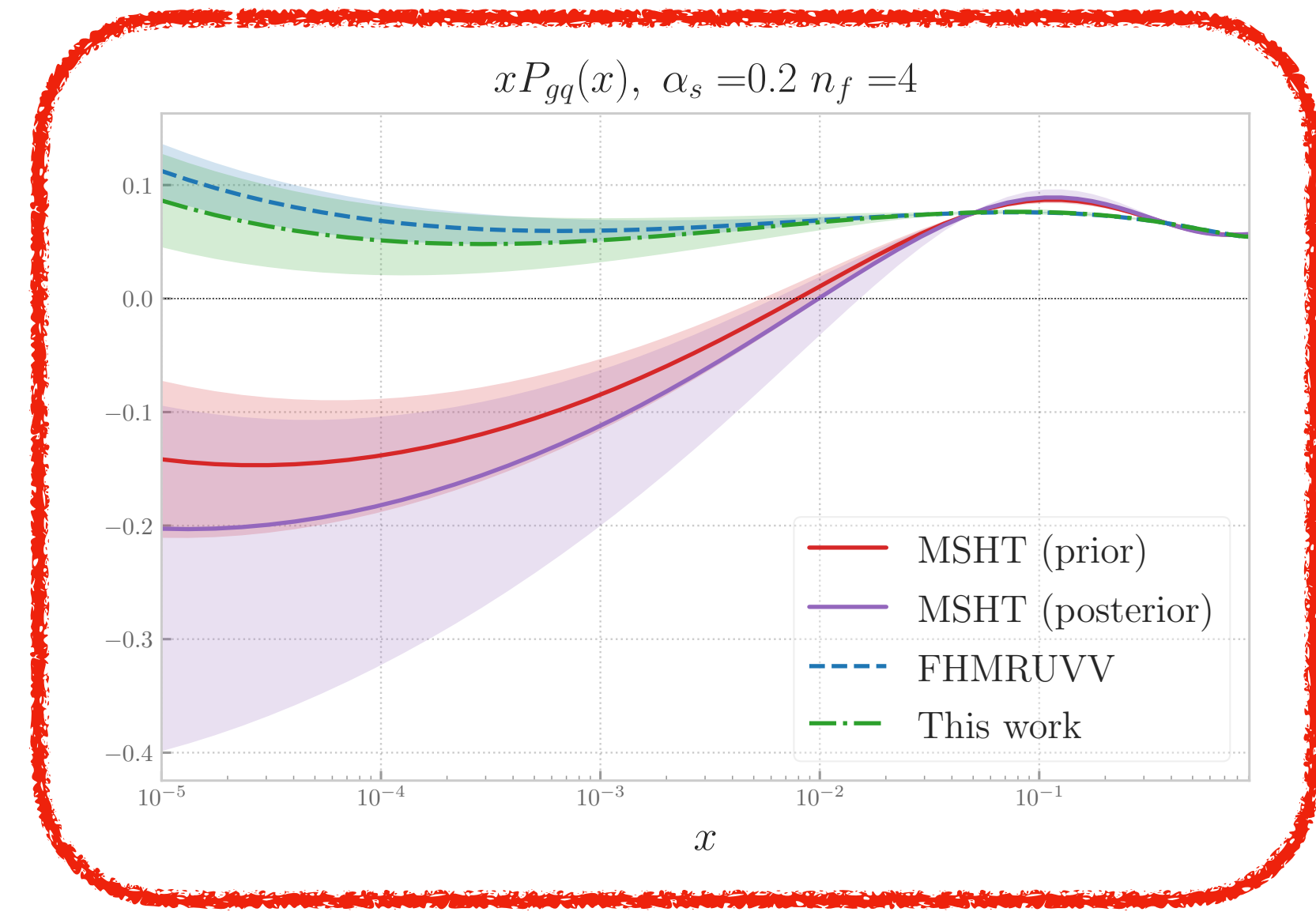
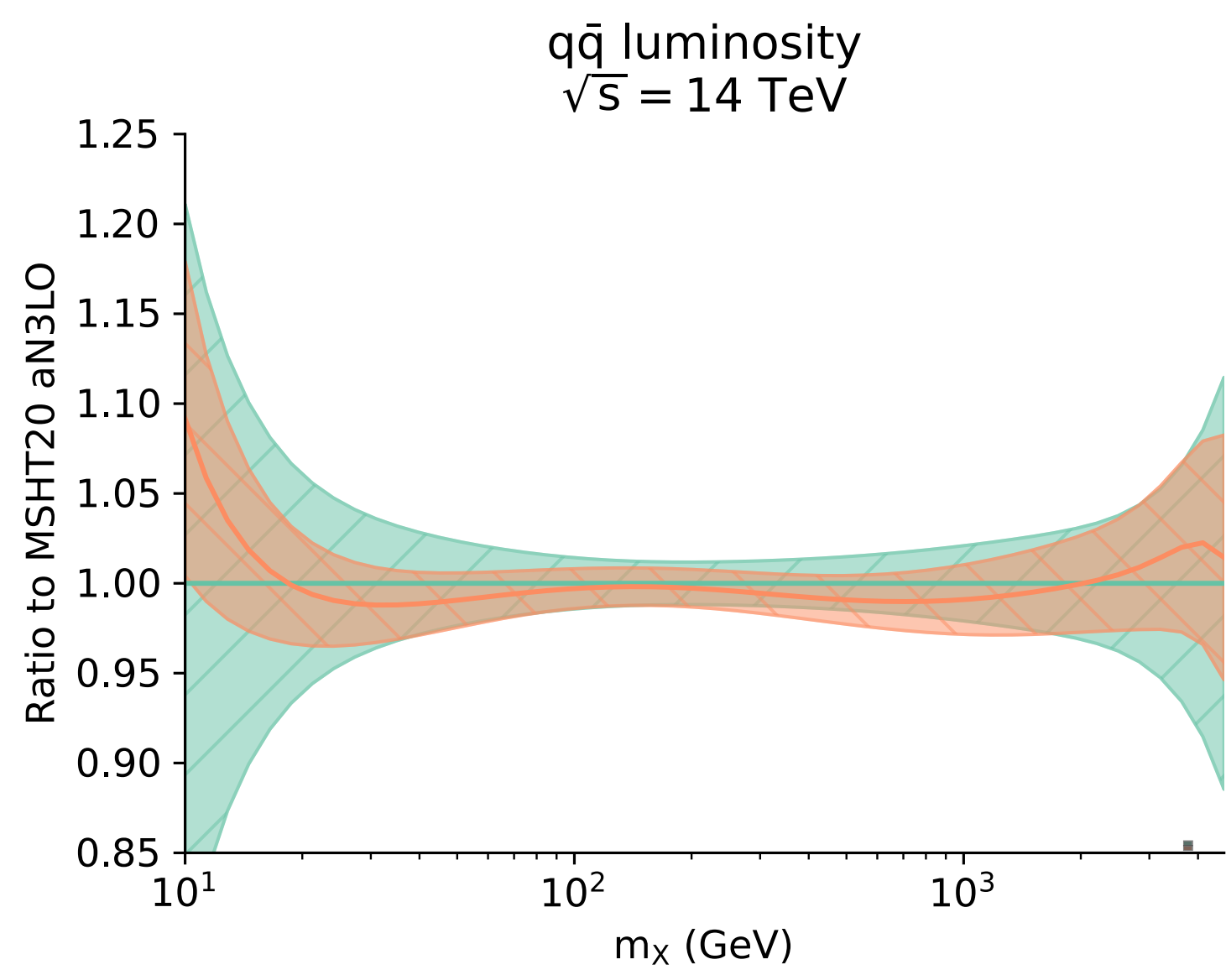
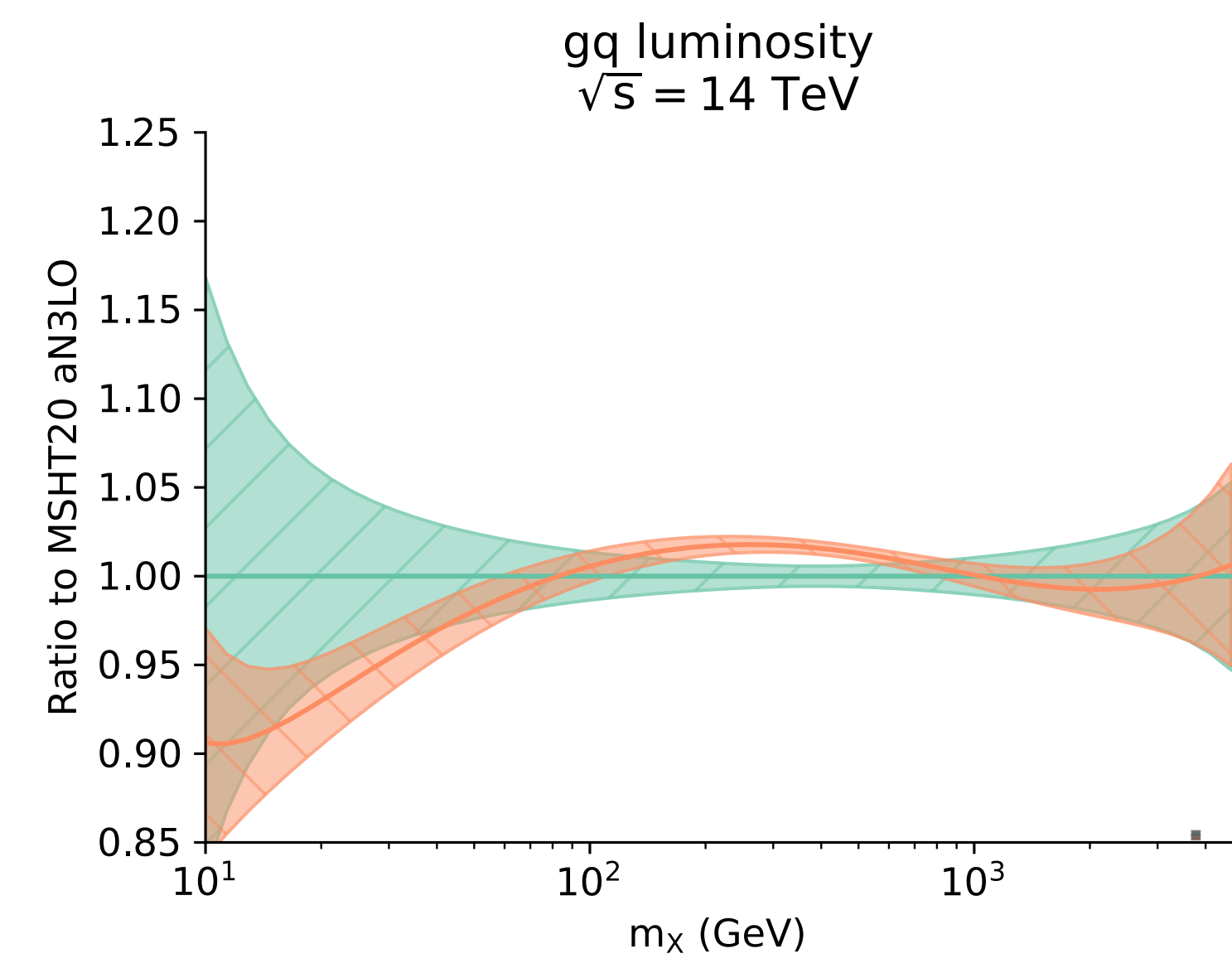
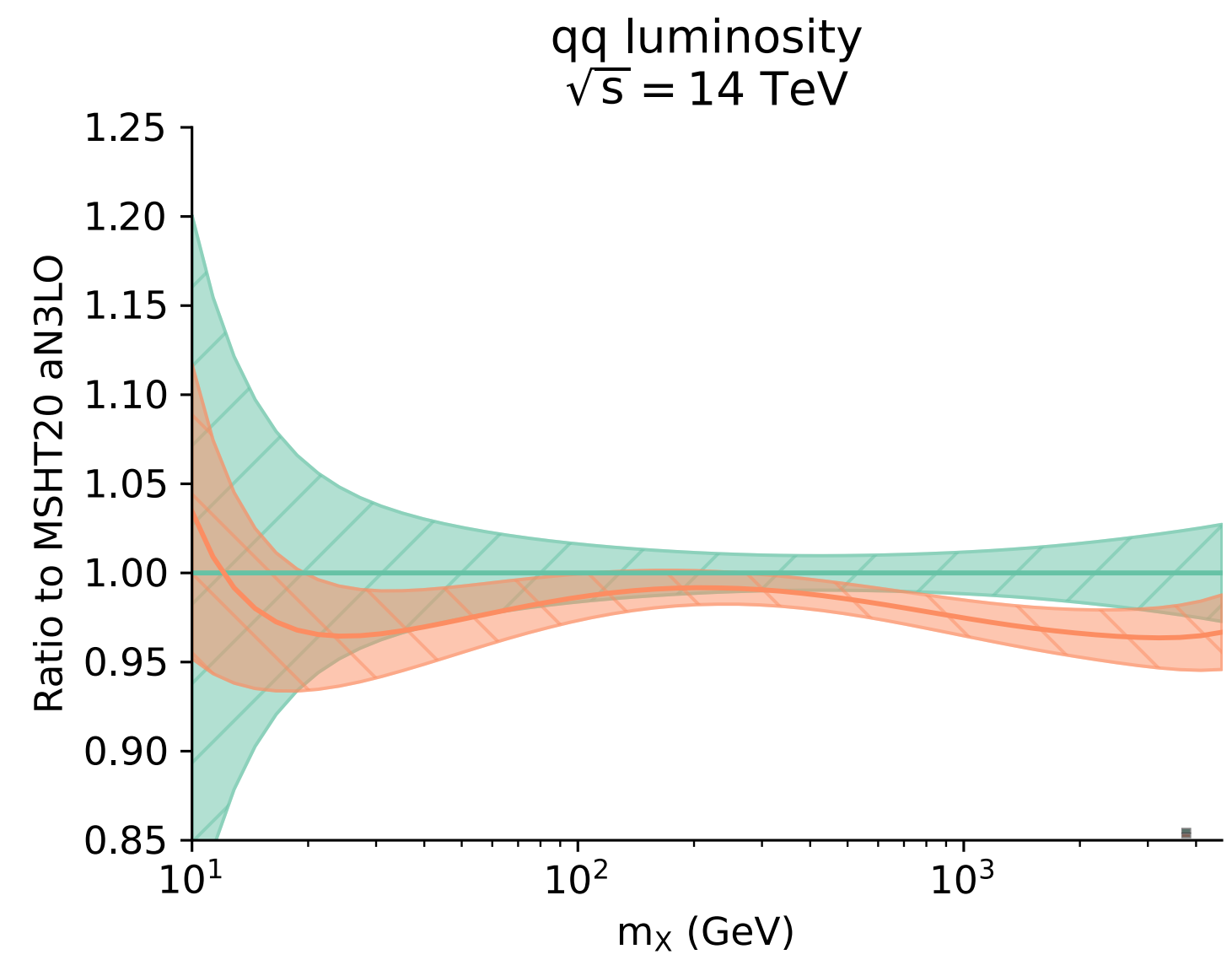
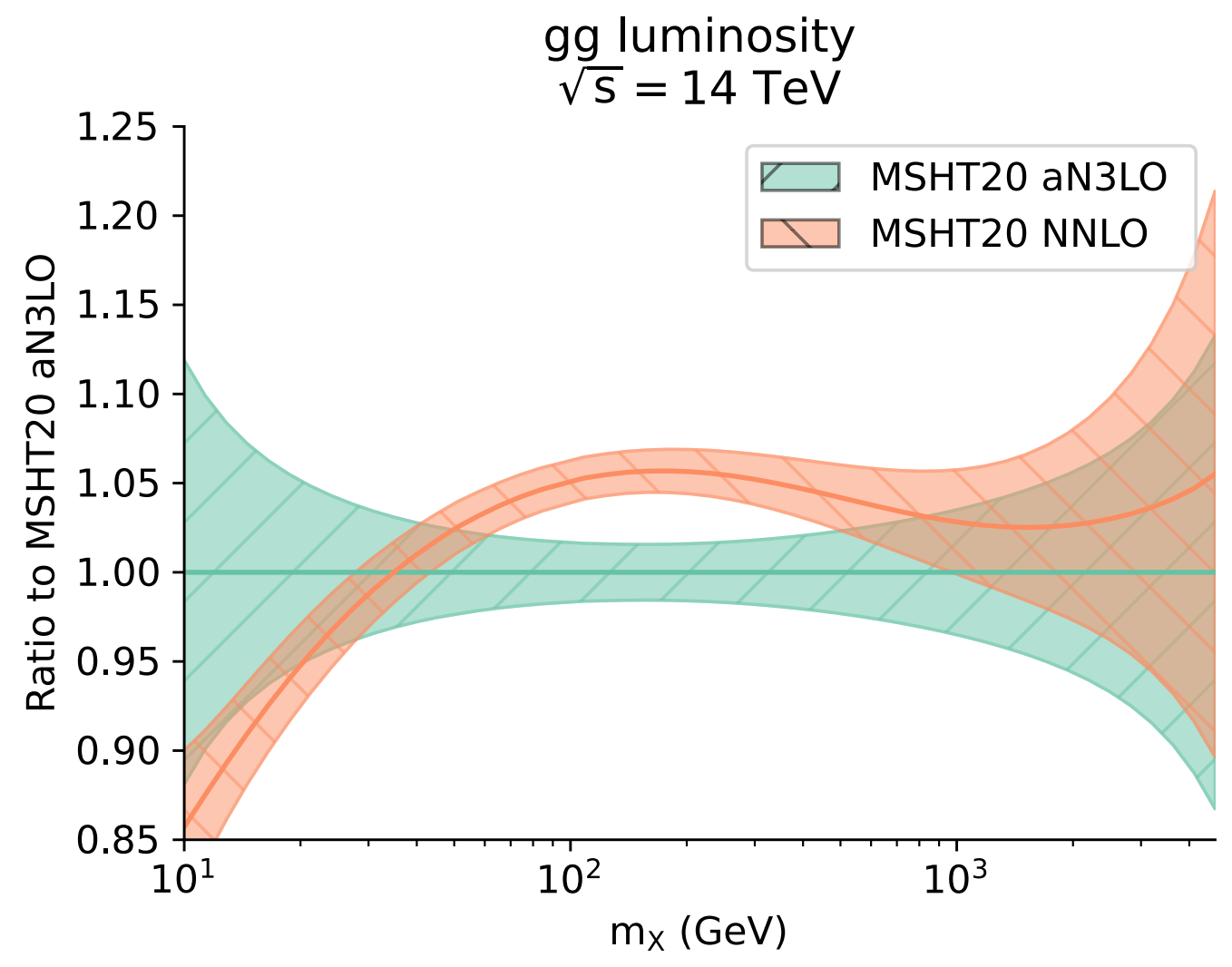
MHOU

Fit quality



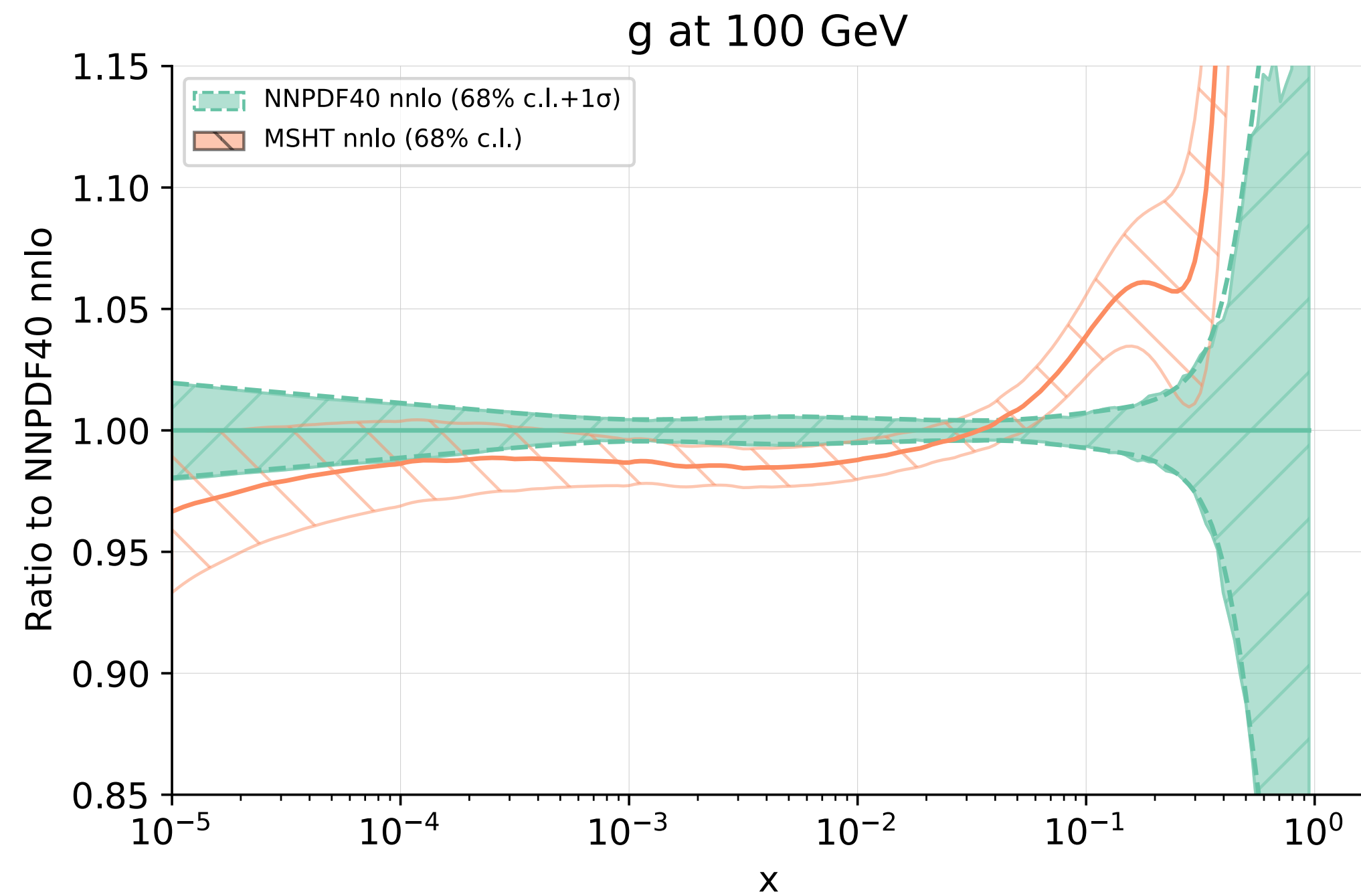
- Without MHOUs fit quality improves as perturbative order improves
- With MHOUs fit quality is independent on perturbative order within χ^2 uncertainties
 $N_{\text{dat}} = 4462, \quad \sigma_{\chi^2} = 0.03$

aN3LO partonic luminosity (MSHT)

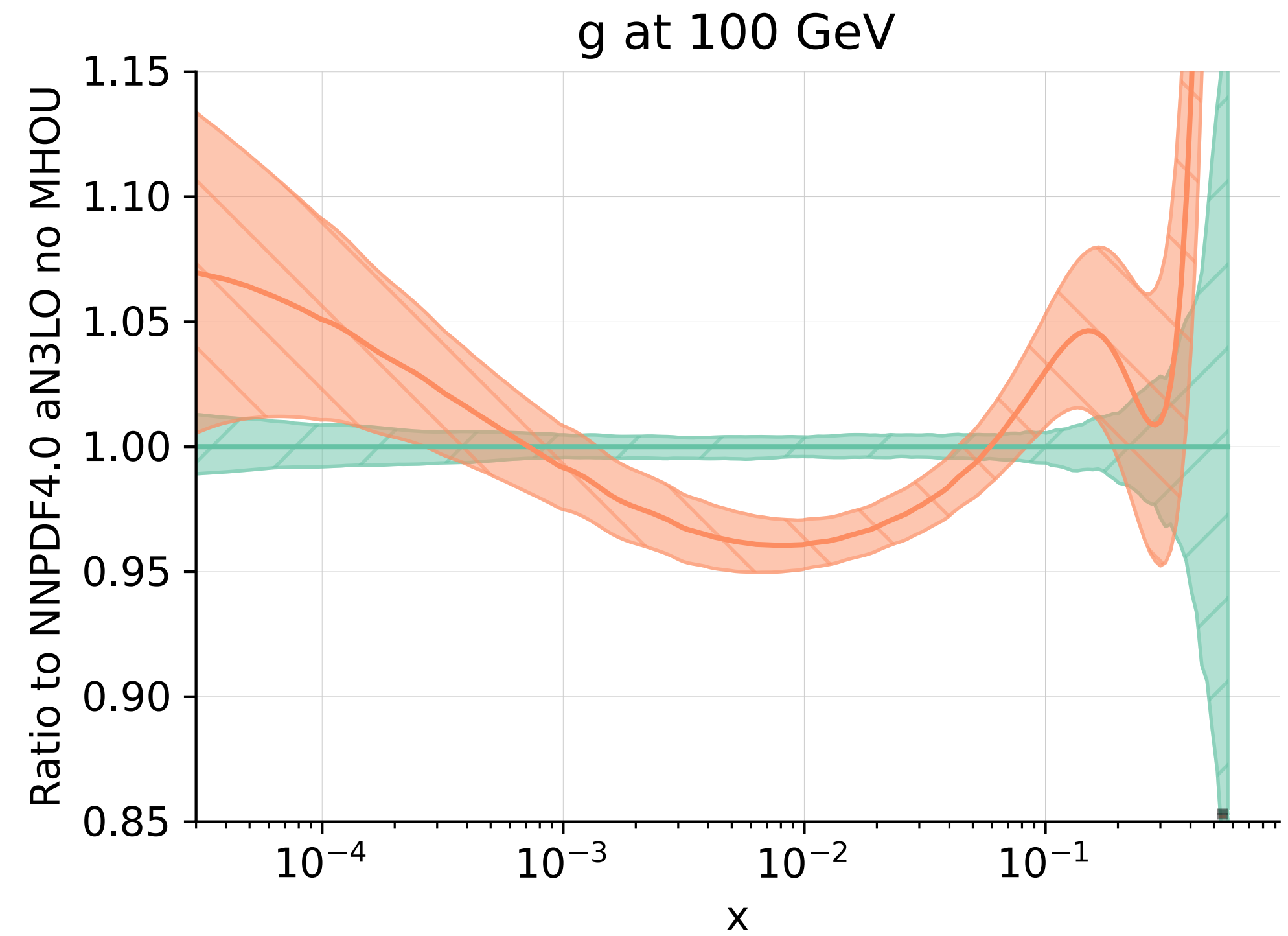


Comparison with MSHT

- Differences similar to those observed at NNLO
- At aN3LO differences in the gluon become bigger
- Most likely due to the differences in P_{gq}



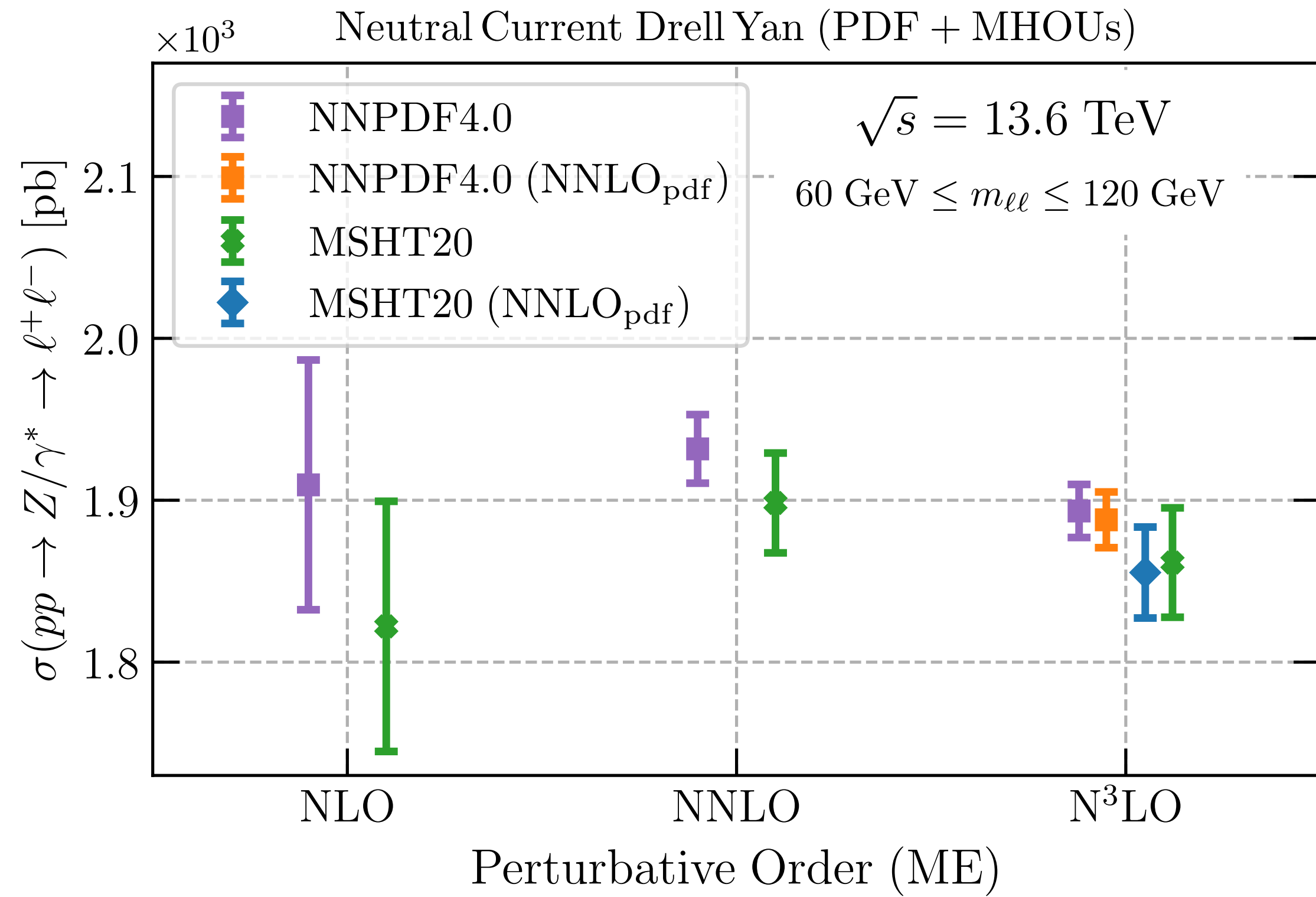
NNLO



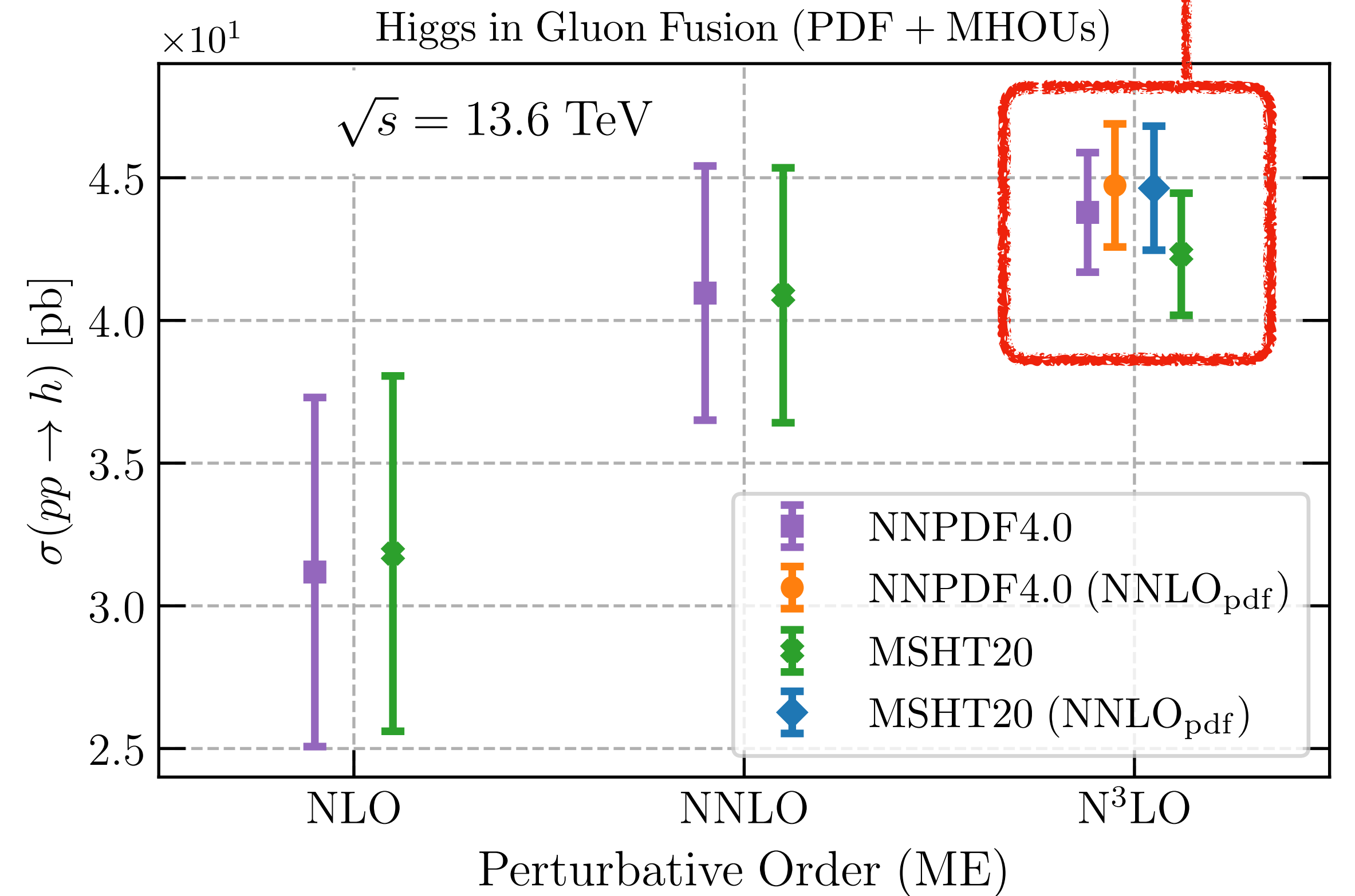
aN3LO

Some pheno

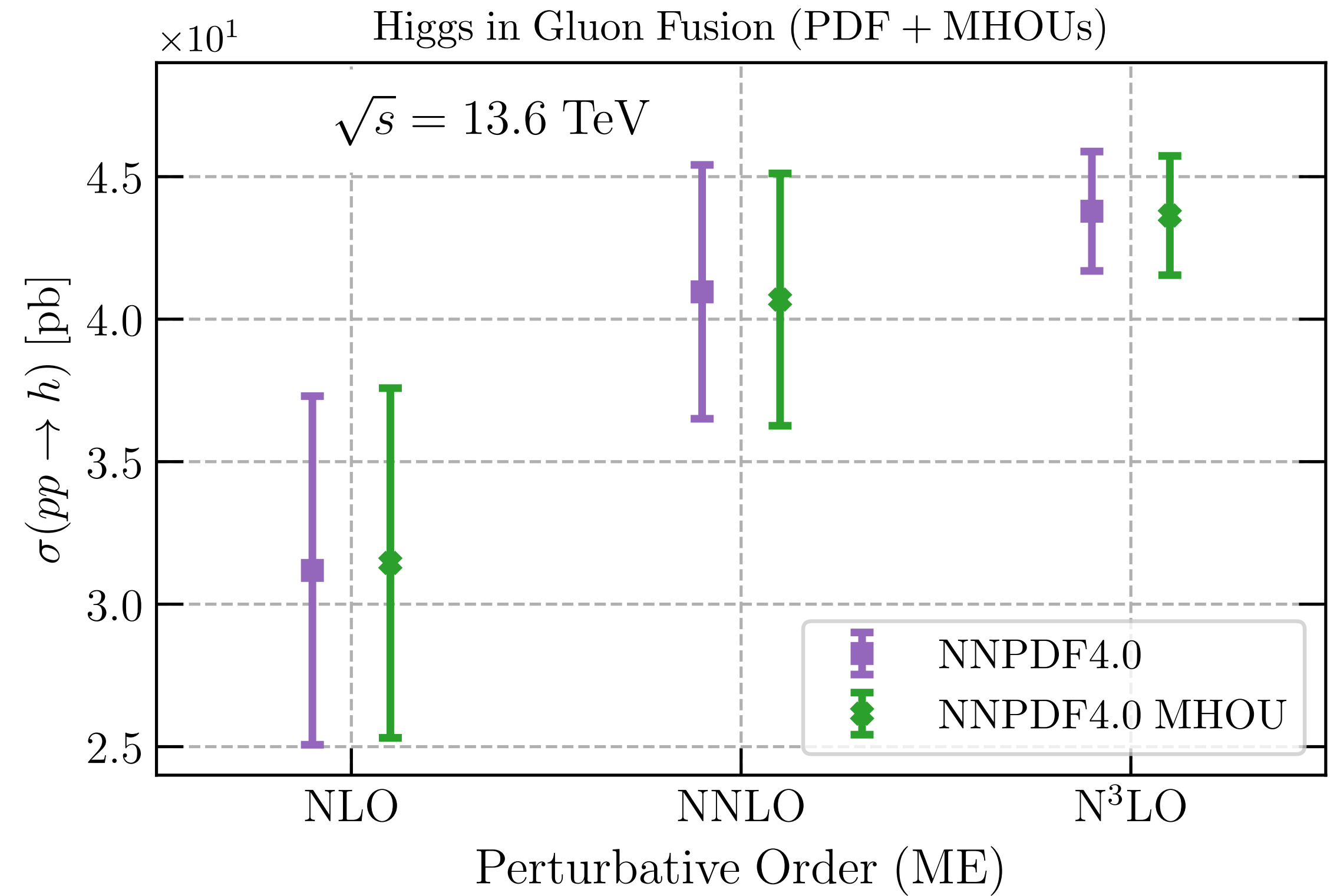
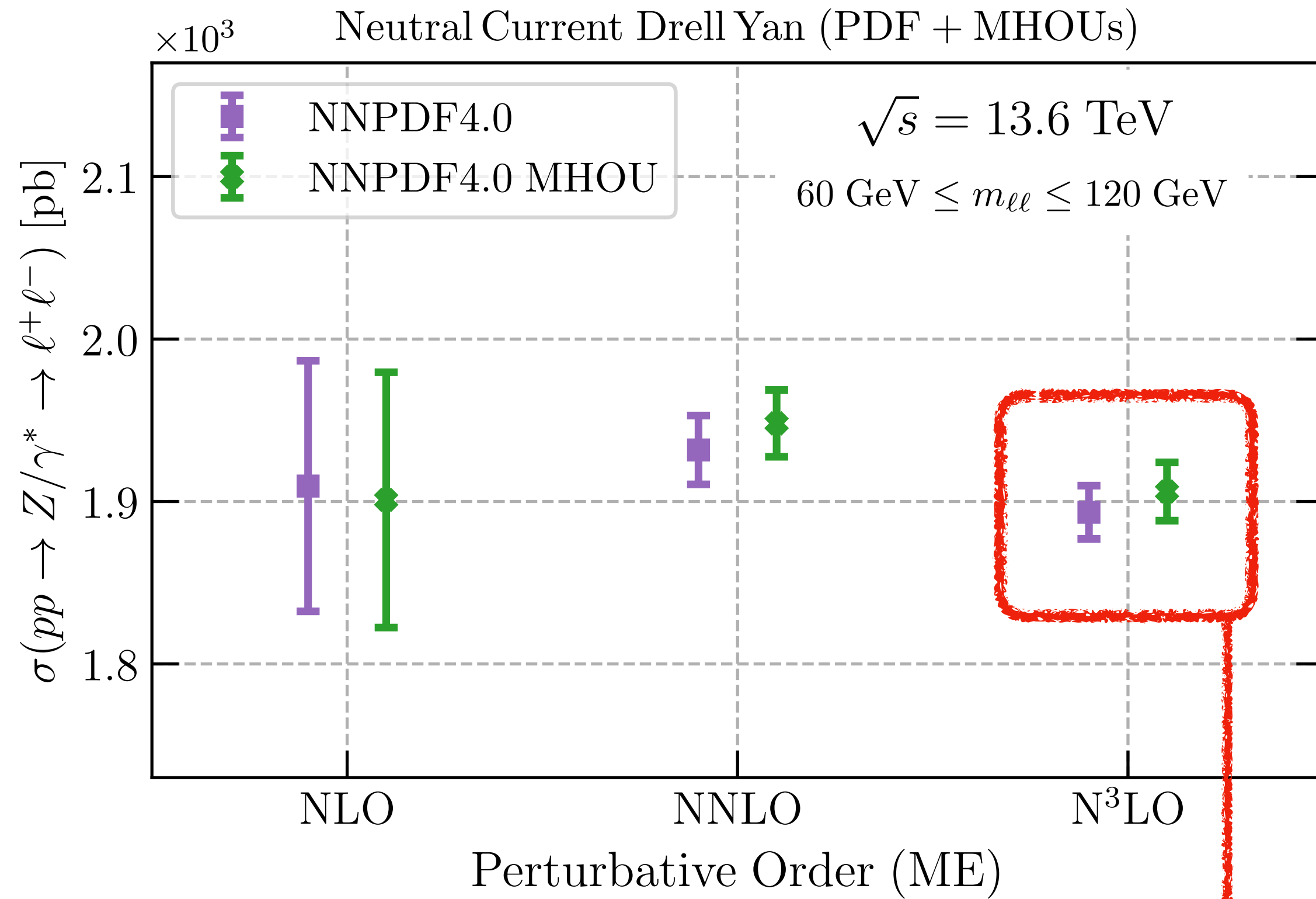
n3loxs [arXiv:2209.06138]



ggHiggs [arXiv:1404.3204]



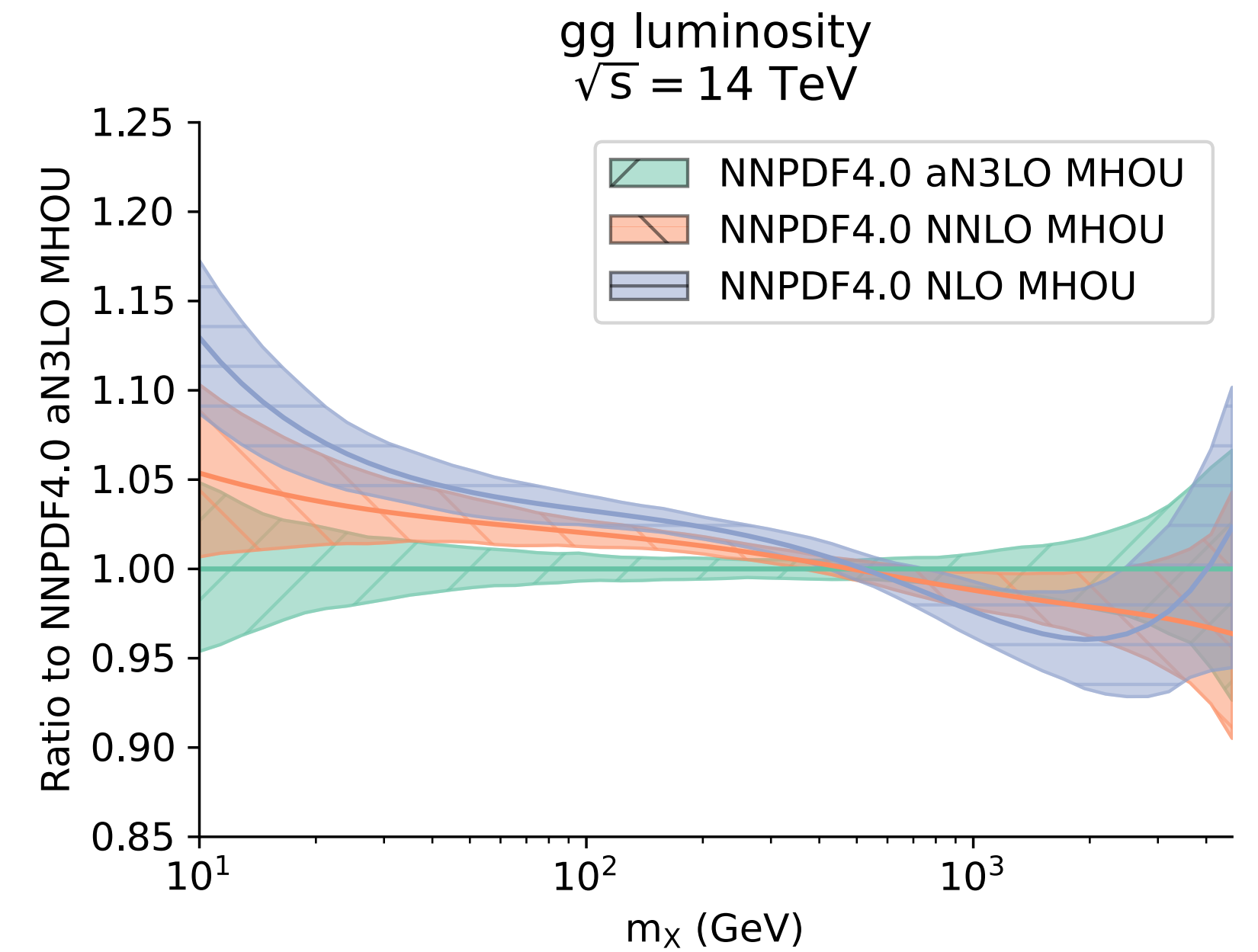
- The predictions based on NNPDF4.0 and MSHT20 compatible within uncertainties (agreement improves at N3LO for DY)
- Using correctly aN3LO PDF in N3LO prediction rather than NNLO PDF reduces difference wrt NNLO result



When considering MHOUs in the fit the differences are small but compared to the total uncertainty (DY)

Summary

- **Two aN3LO PDF sets** are now available (MSHT and NNPDF)
- NNPDF4.0 aN3LO released **both with and without MHO**
- **IHO** are estimated and included in the analysis (different approaches followed by NNPDF and MSHT)
- **Impact of N3LO corrections** found by MSHT and NNPDF seems consistent, with differences which can be traced back to different input splitting functions
- Biggest differences when going from NNLO to aN3LO are observed in the **gluon**



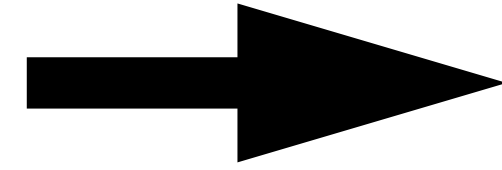
Thank you!

Backup slides

Nonsinglet

$$P_{ns,\pm}^{(3)} \quad P_{ns,s}^{(3)}$$

Nonsinglet splitting functions can be determined with high accuracy



- 8 moments [[JHEP 10 \(2017\) 041](#)]
- Small- x limit [[JHEP 08 \(2022\) 135](#)]
- Large- x limit [[JHEP 10 \(2017\) 041](#)]
- large- n_f limit, i.e. $\mathcal{O}(n_f^2)$, $\mathcal{O}(n_f^3)$ [[Nucl. Phys. B 915 \(2017\) 335–362](#)]

What we know

Our approximation

$$\gamma_{ij}^{(3,0)} + n_f \gamma_{ij}^{(3,1)} + n_f^2 \gamma_{ij}^{(3,2)} + n_f^3 \gamma_{ij}^{(3,3)}$$

$$\gamma_{ij}^{(3)}(N) = \underbrace{\gamma_{ij,n_f}^{(3)}(N)}_{\text{large-}n_f \text{ limit}} + \underbrace{\gamma_{ij,N \rightarrow \infty}^{(3)}(N)}_{\text{large-}x \text{ limit}} + \underbrace{\gamma_{ij,N \rightarrow 0}^{(3)}(N)}_{\text{small-}x \text{ limit}} + \underbrace{\tilde{\gamma}_{ij}^{(3)}(N)}_{\text{remainder}}$$

large- n_f limit

large- x limit

small- x limit

remainder

$$A_4 S_1(N) + B_4 + C_4 \frac{S_1(N)}{N} + D_4 \frac{1}{N}$$

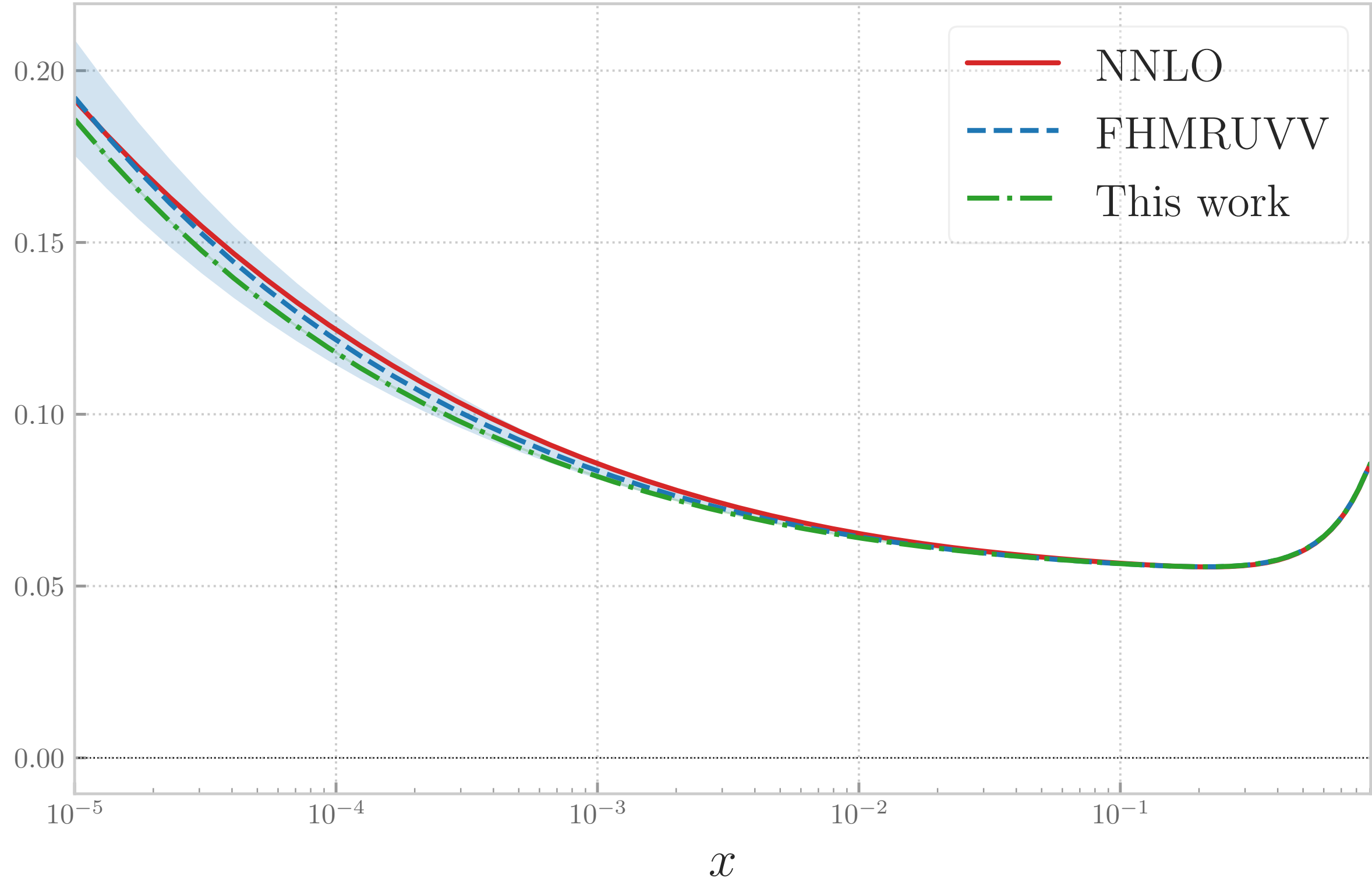
$$\sum_{k=1}^6 c_{ns}^k (-1)^k \frac{k!}{N^{k+1}}$$

Leading unknown contributions
(small- N and large- N)

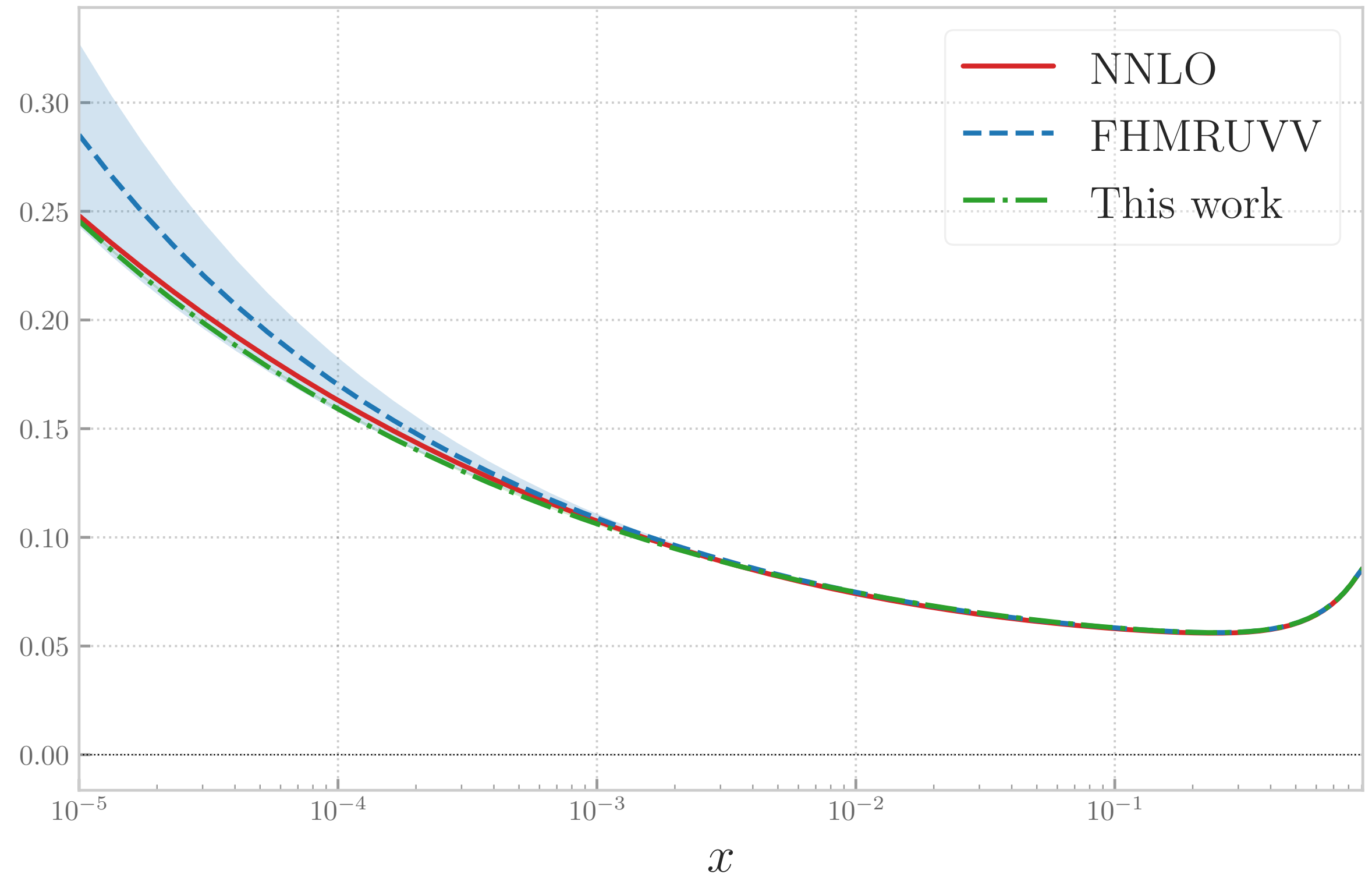
$G_1^{\text{ns},\pm}(N)$	1
$G_2^{\text{ns},\pm}(N)$	$\mathcal{M}[(1-x) \ln(1-x)](N)$
$G_3^{\text{ns},\pm}(N)$	$\mathcal{M}[(1-x) \ln^2(1-x)](N)$
$G_4^{\text{ns},\pm}(N)$	$\mathcal{M}[(1-x) \ln^3(1-x)](N)$
$G_5^{\text{ns},\pm}(N)$	$\frac{S_1(N)}{N^2}$
$G_6^{\text{ns},\pm}(N)$	$\frac{1}{(N+1)^2}$
$G_7^{\text{ns},\pm}(N)$	$\frac{1}{(N+1)^3}$
$G_8^{\text{ns},+}(N), G_8^{\text{ns},-}(N)$	$\frac{1}{(N+2)}, \frac{1}{(N+3)}$

Nonsinglet: comparison to previous approximation

$$(1-x)P_{NS,+}(x), \alpha_s = 0.2 \ n_f = 4$$



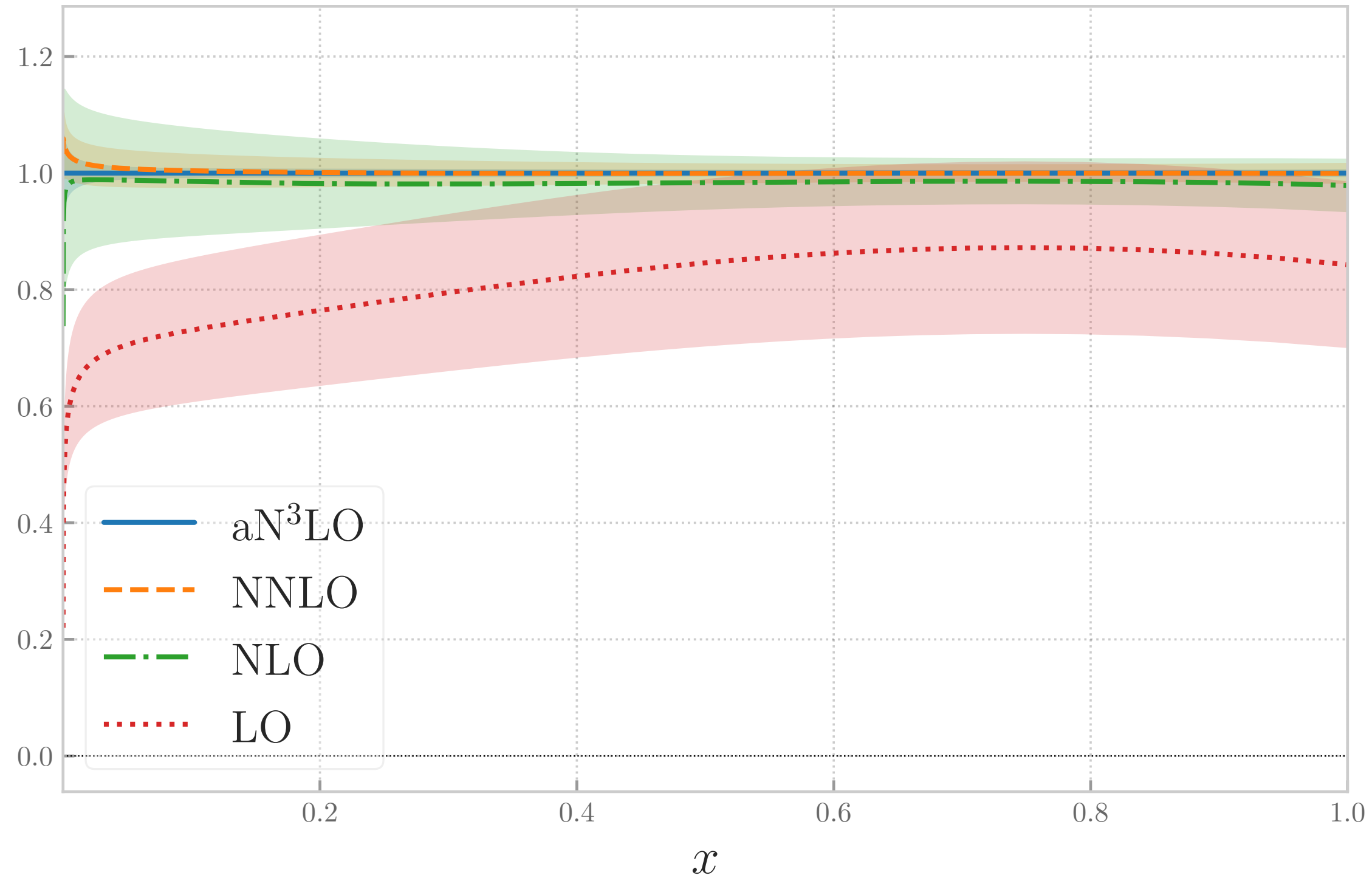
$$(1-x)P_{NS,-}(x), \alpha_s = 0.2 \ n_f = 4$$



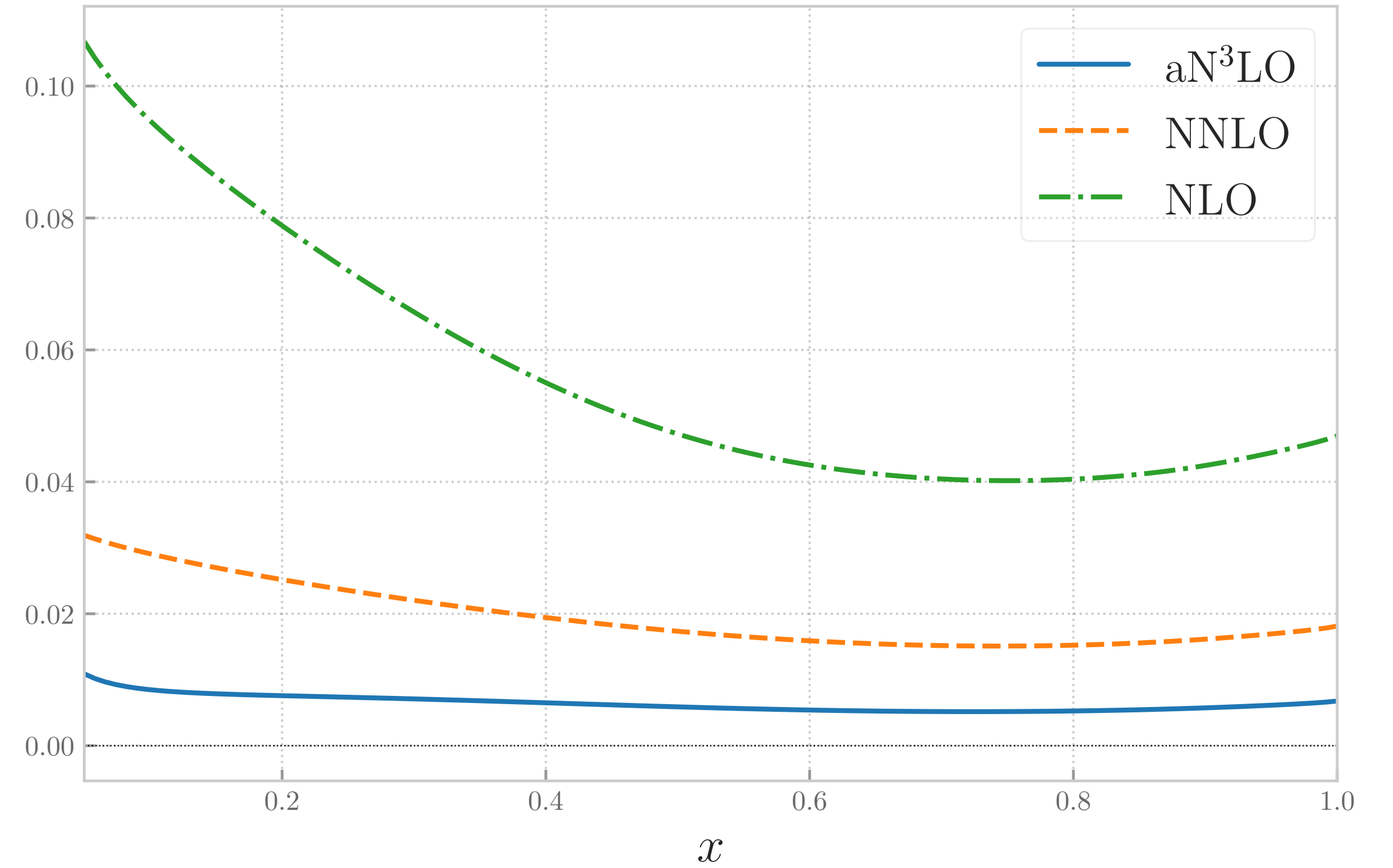
- Compatibility with previous approximation FHMRUVV [[JHEP 10 \(2017\) 041](#)]
- Additional small- x info included in current approximation

Nonsinglet: LO, NLO, NNLO, aN3LO with MHOU

$$P_{NS,+}(x), \alpha_s = 0.2 \ n_f = 4$$

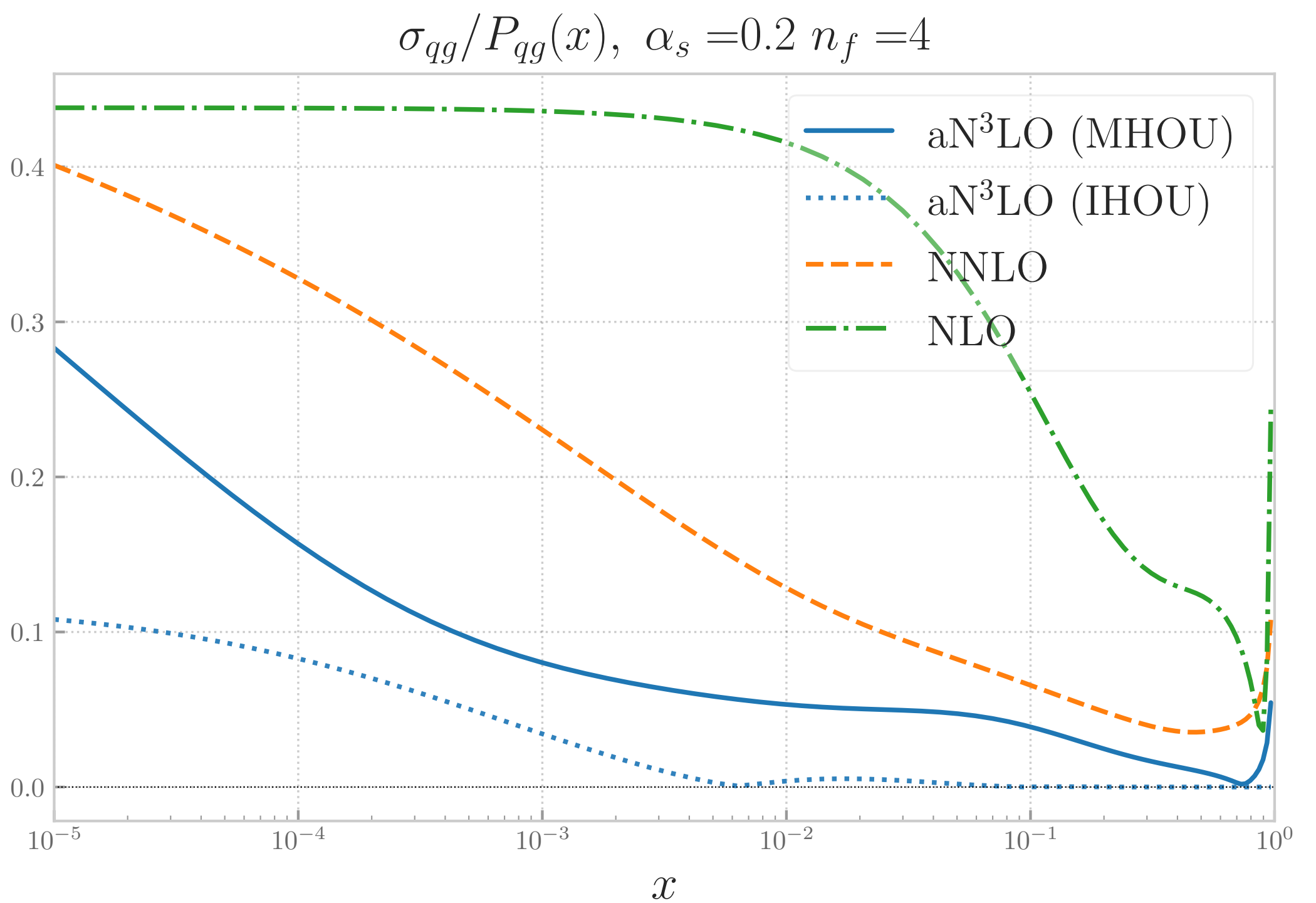
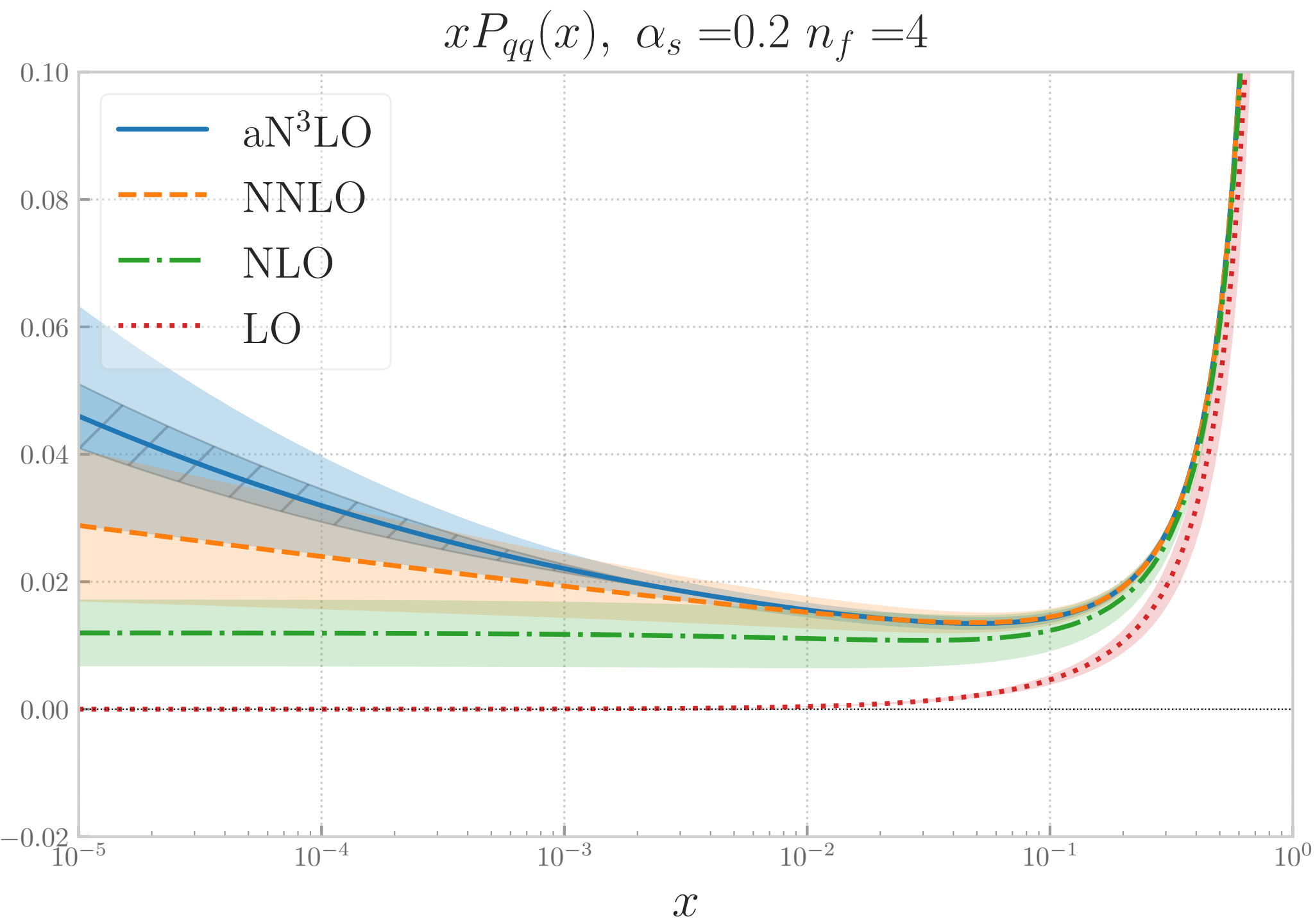


$$\sigma_{NS,+}/P_{NS,+}(x), \alpha_s = 0.2 \ n_f = 4$$



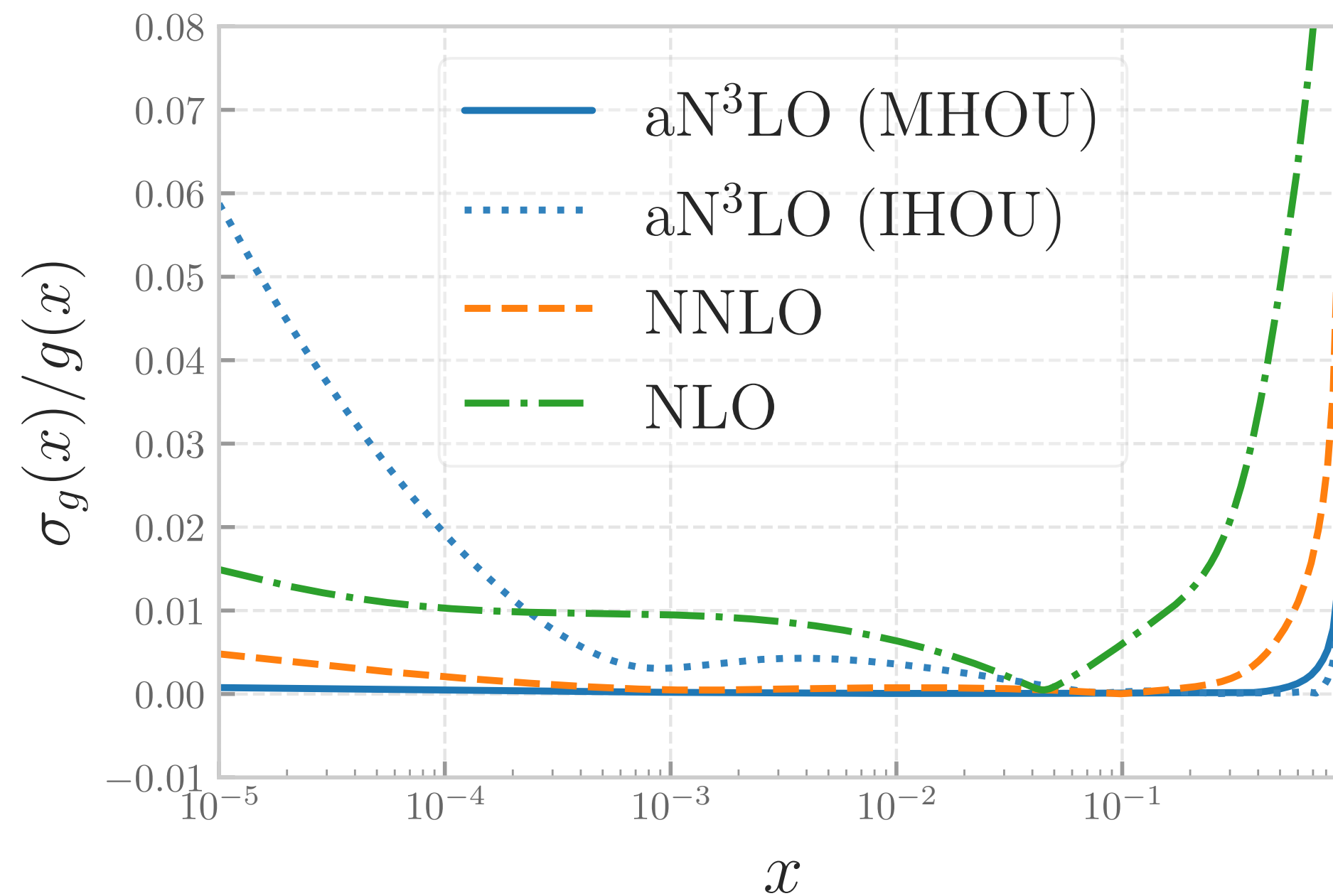
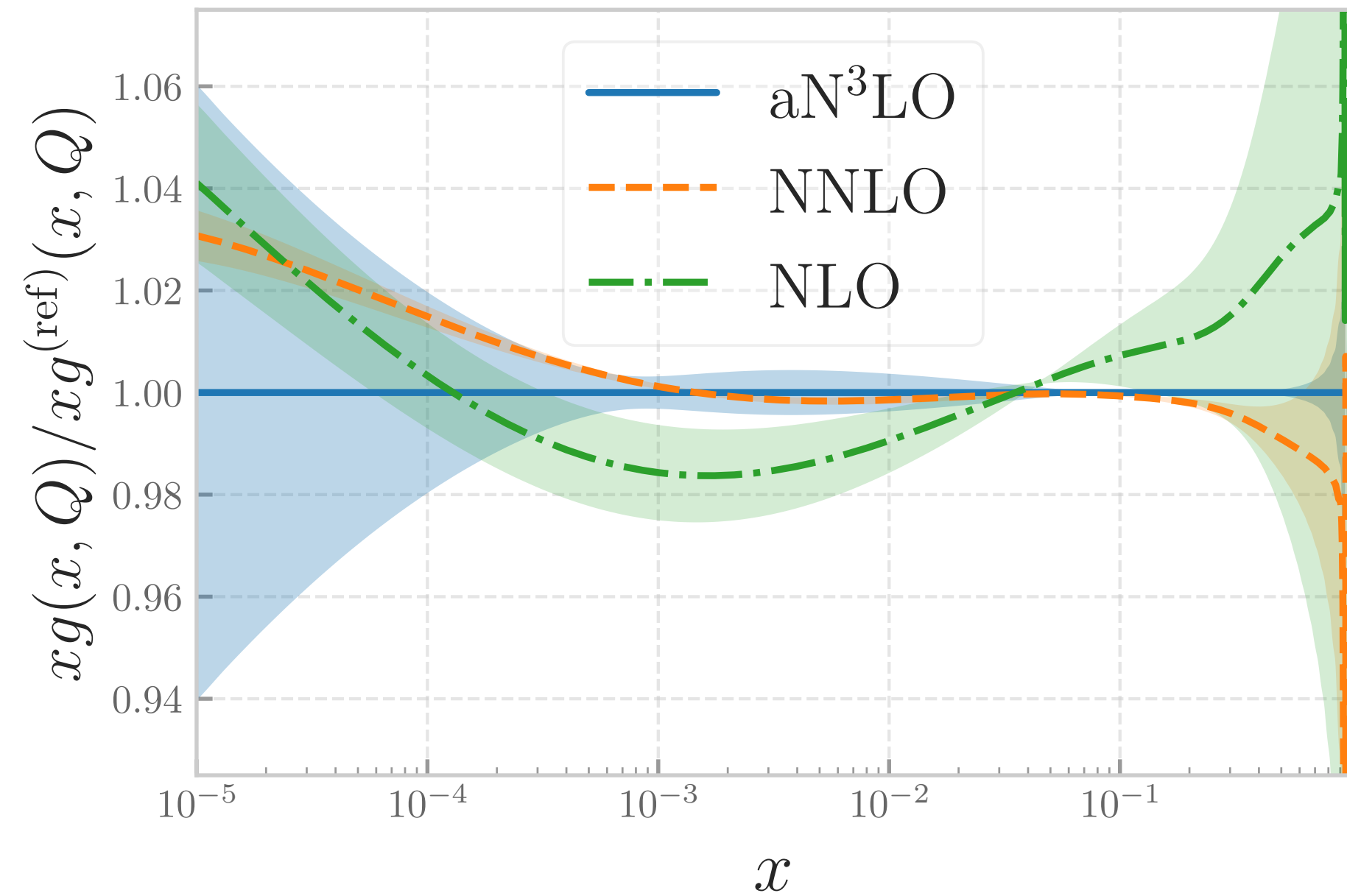
- Good perturbative convergence
- Except at small- x (where nonsinglet sector is subdominant anyway) N³LO corrections are negligible
- Current approximation can be considered exact (no IHOU) with negligible MHOU

singlet: small-x



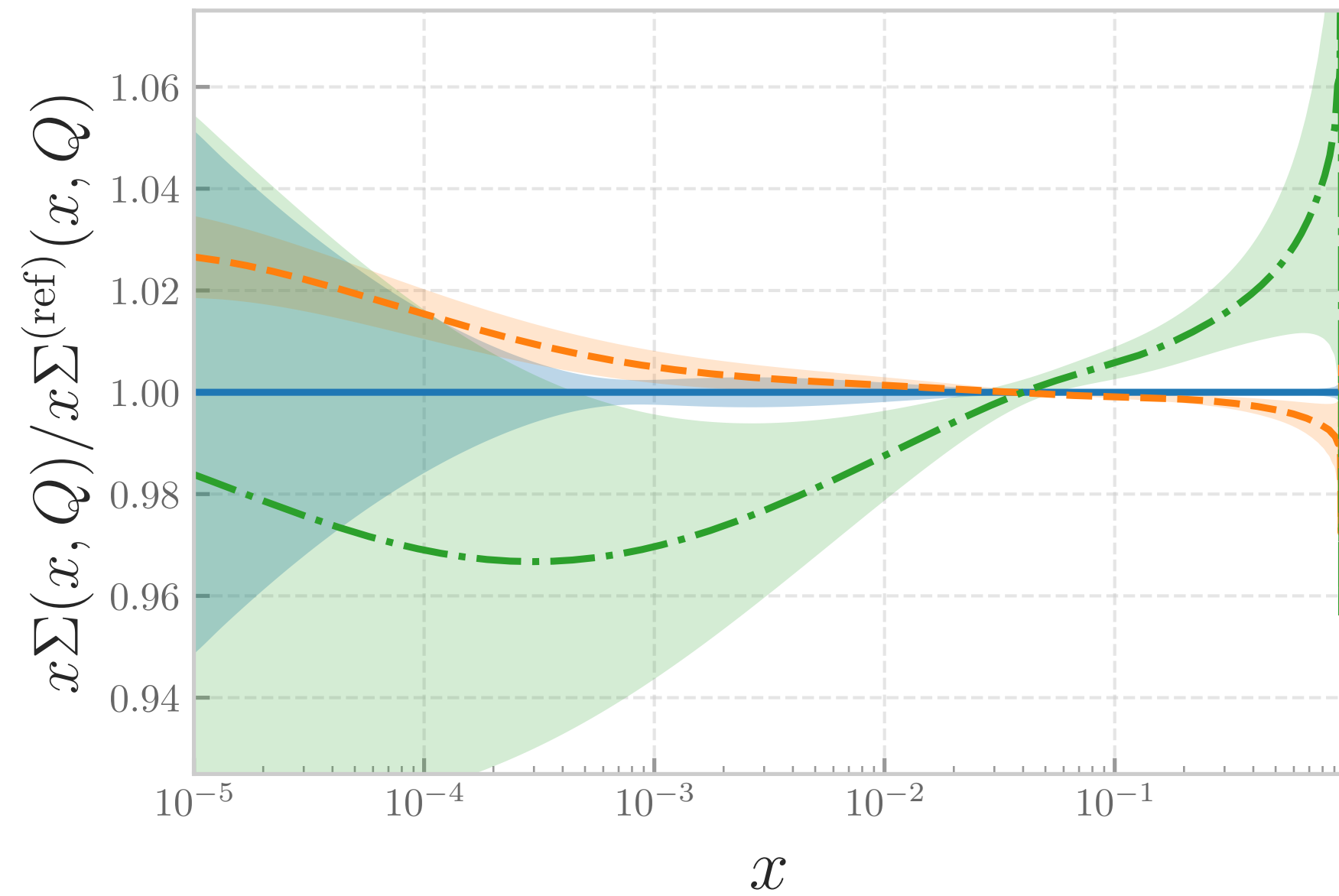
- aN3LO and NNLO result agree within uncertainties
- aN3LO uncertainties are sizeable
- MHO (IHO) dominate the quark (gluon) sector

aN3LO evolution

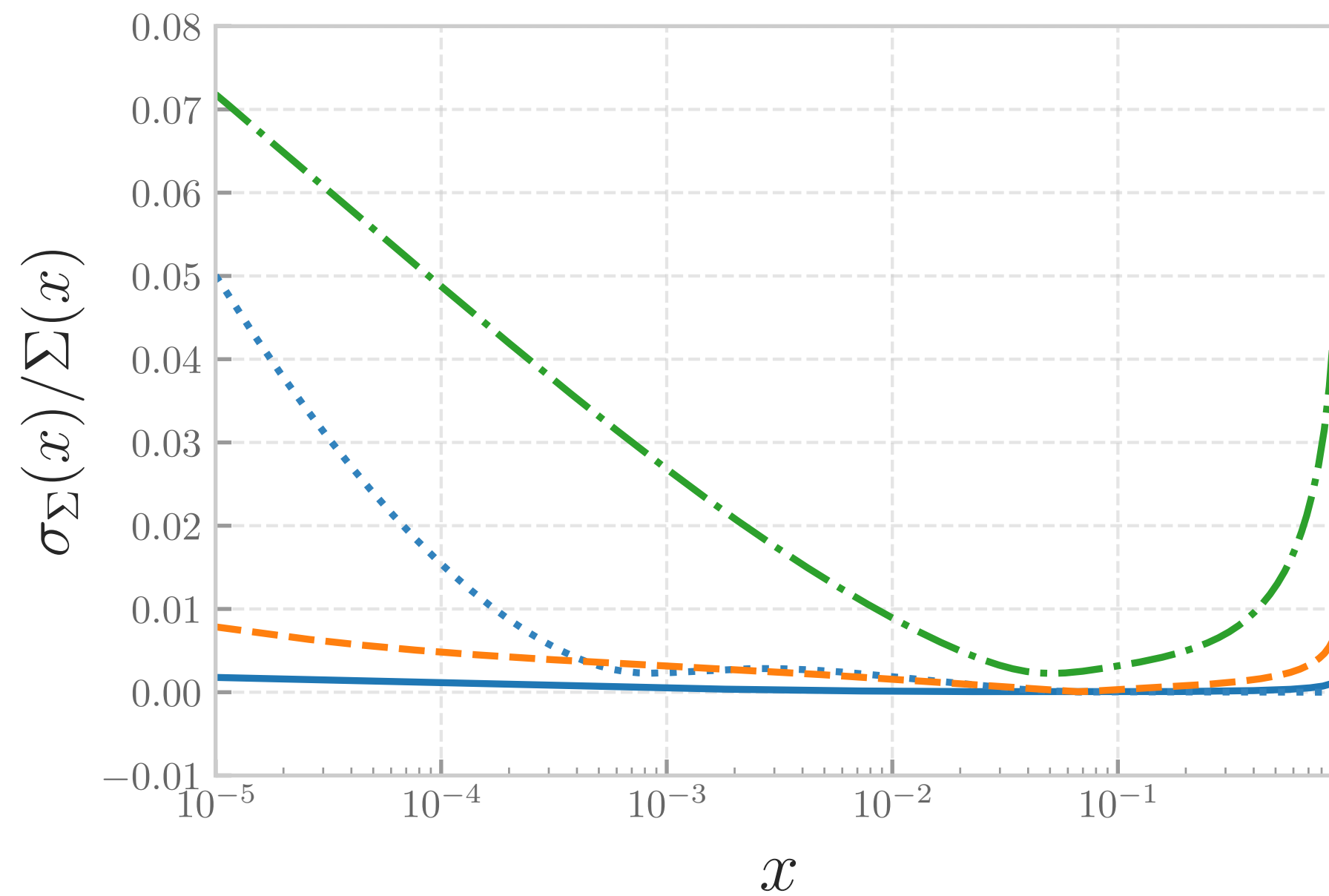


- comparison between NLO, NNLO and aN3LO evolution starting from NNPDF4.0 NNLO at 1.65 GeV and evolving up to 100 GeV
- Almost no difference between aN3LO and NNLO for nonsinglet PDFs
- For $x < 10^{-3}$ singlet evolution is weaker at aN3LO

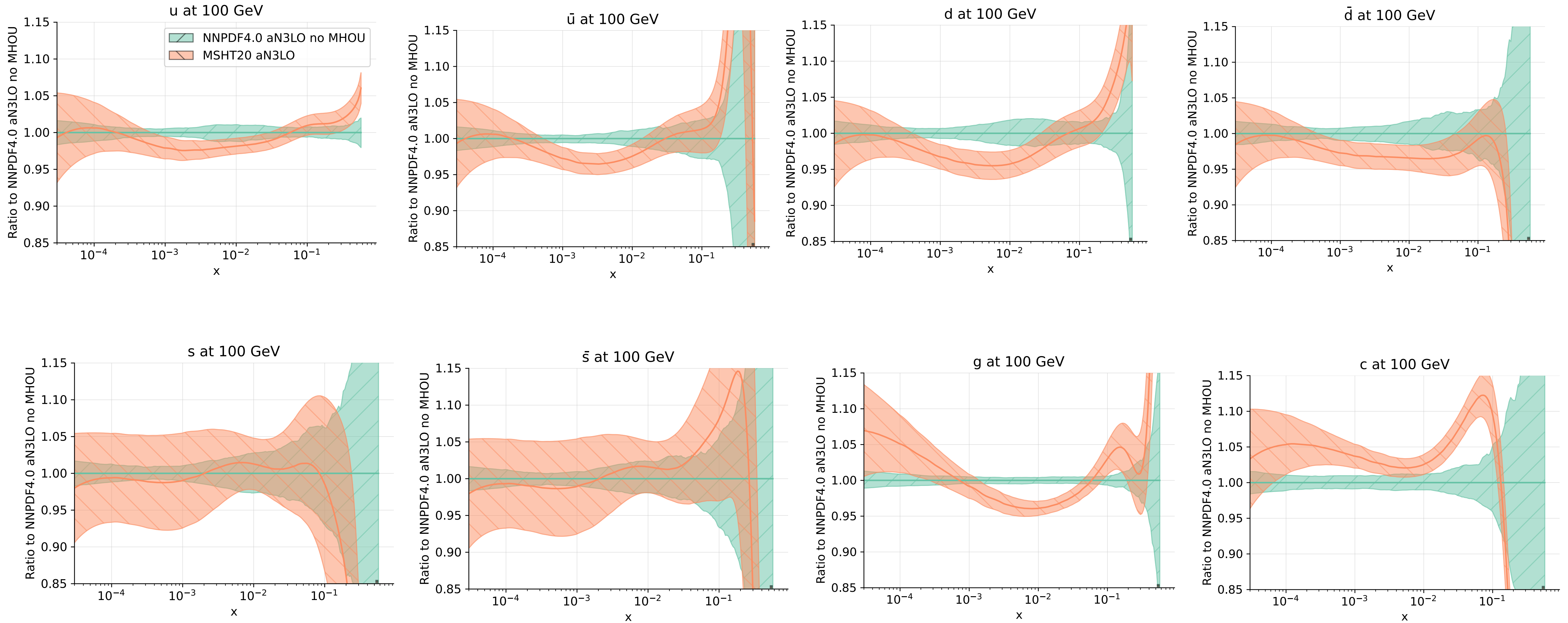
aN3LO evolution



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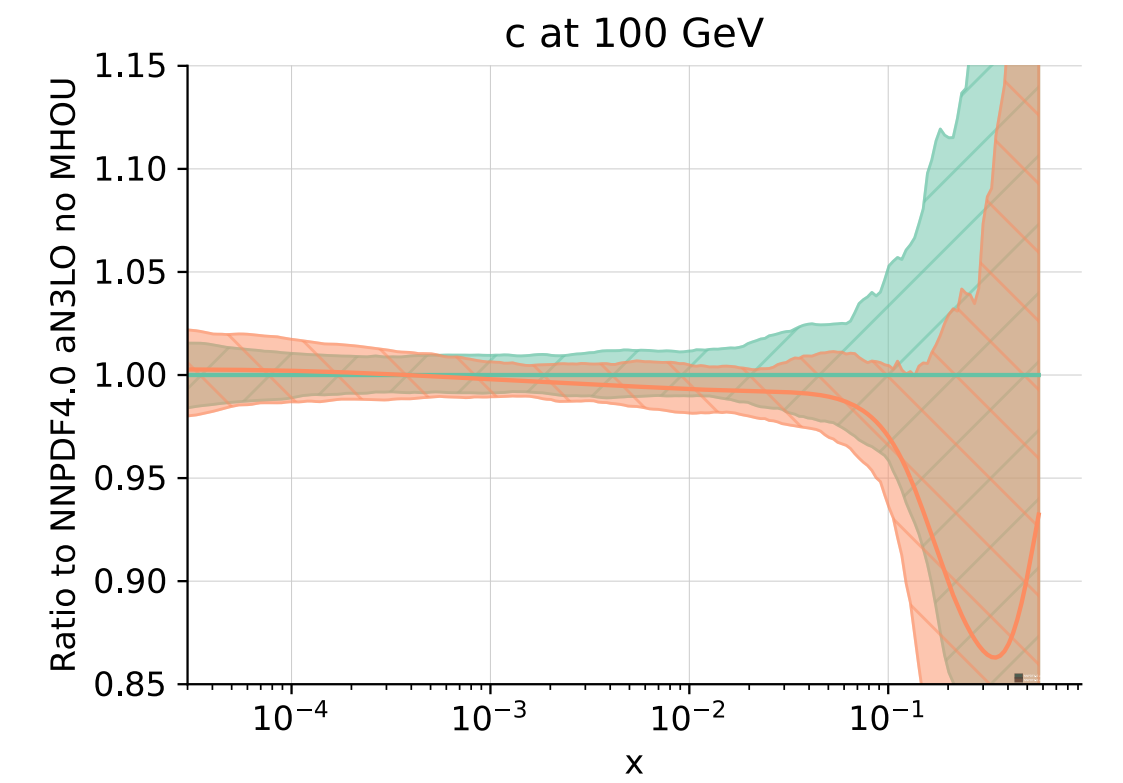
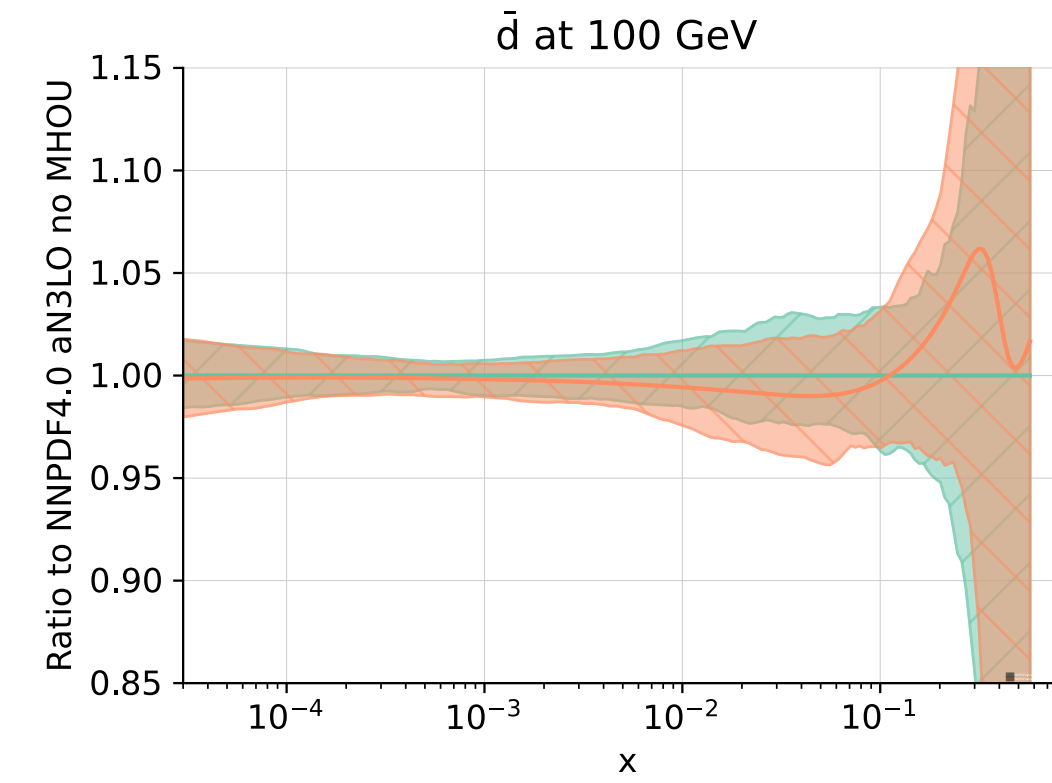
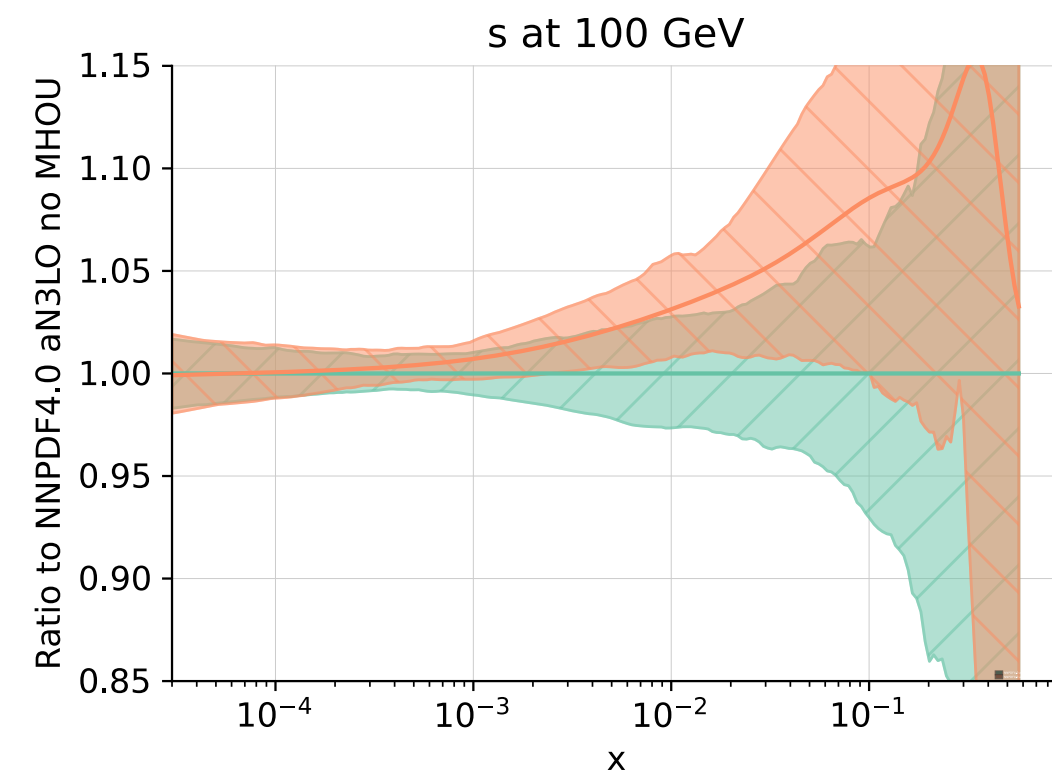
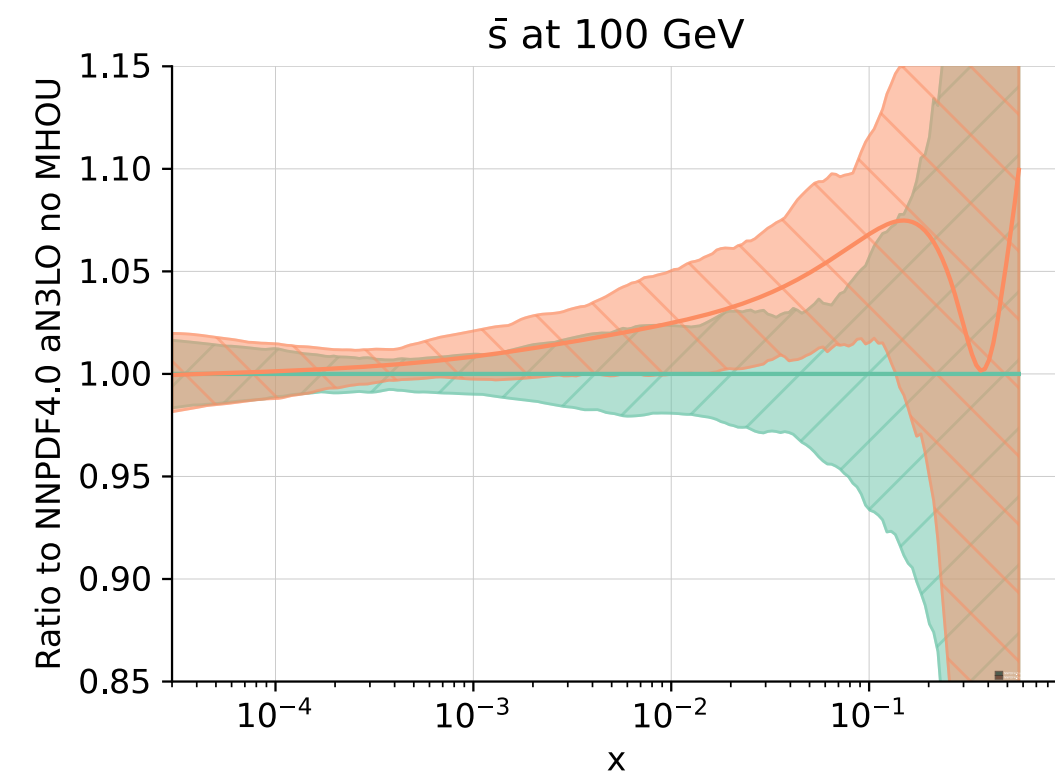
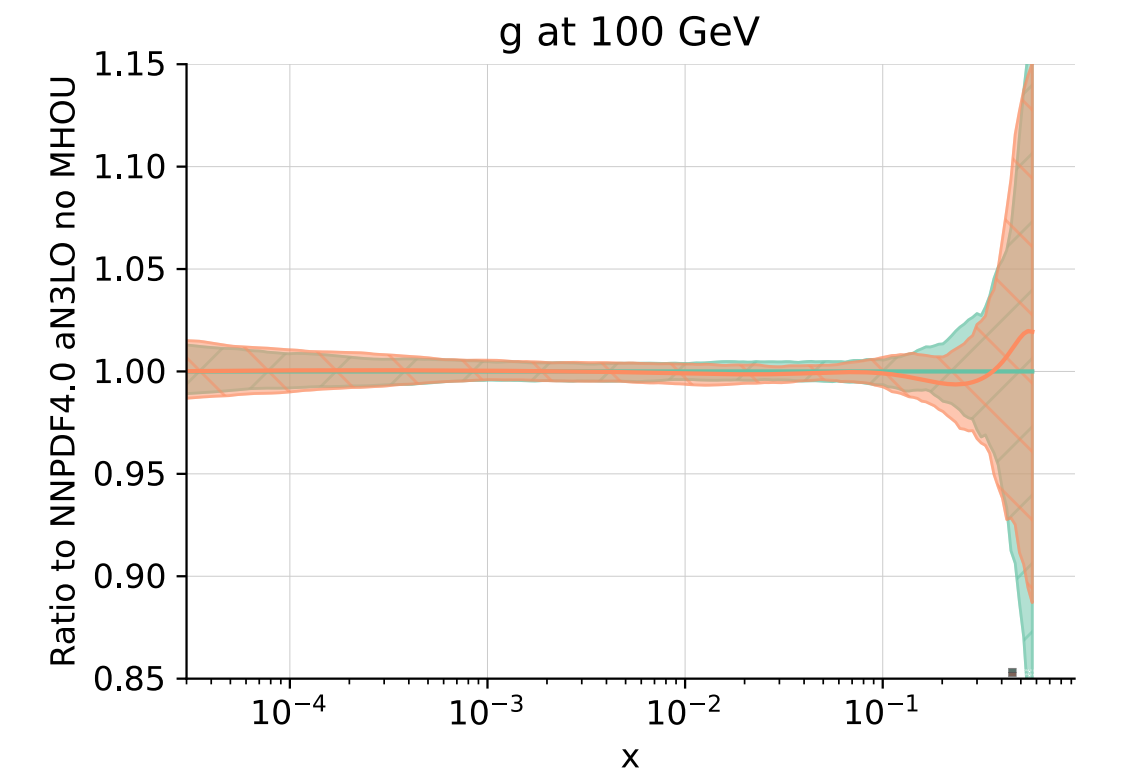
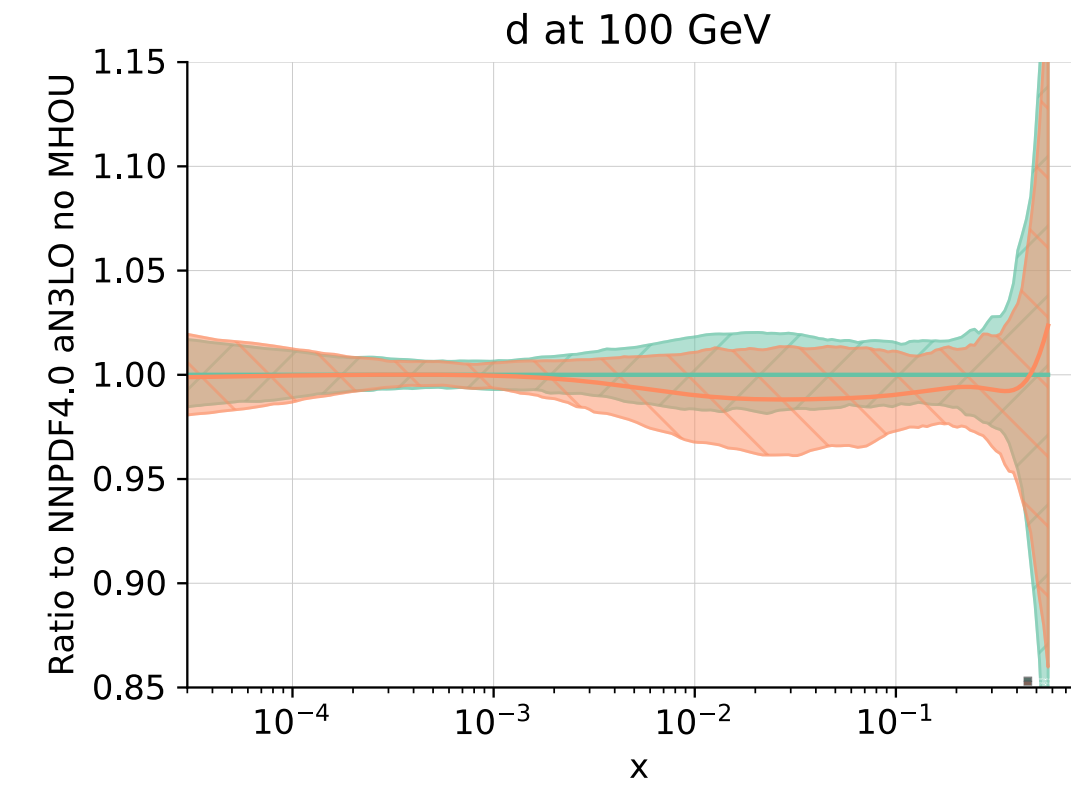
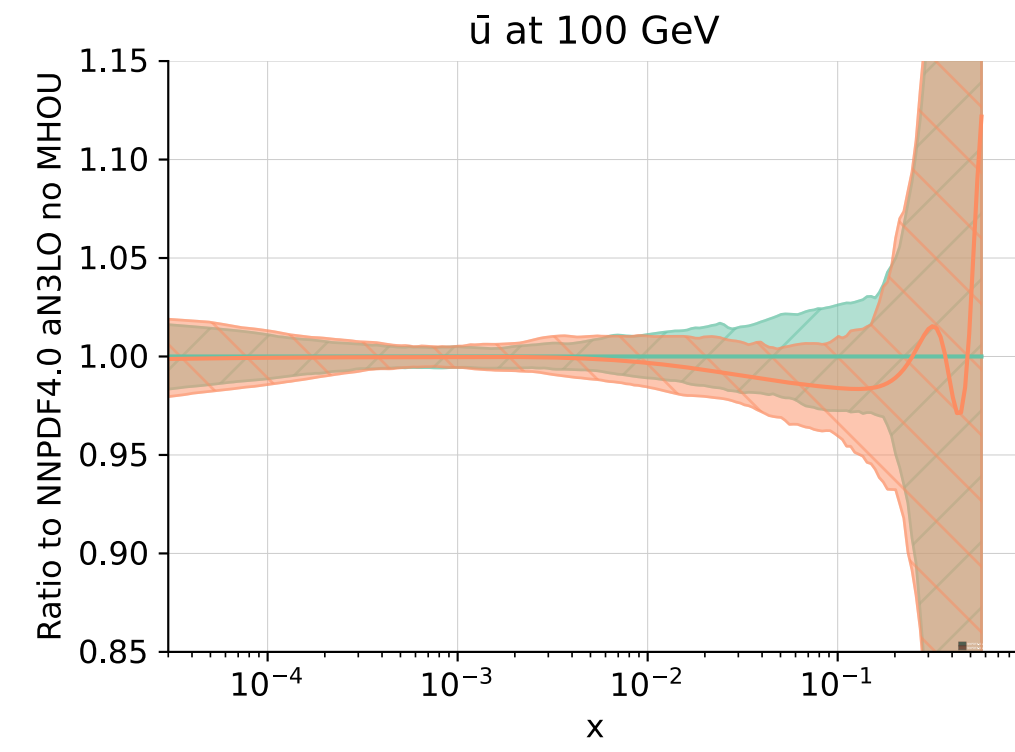
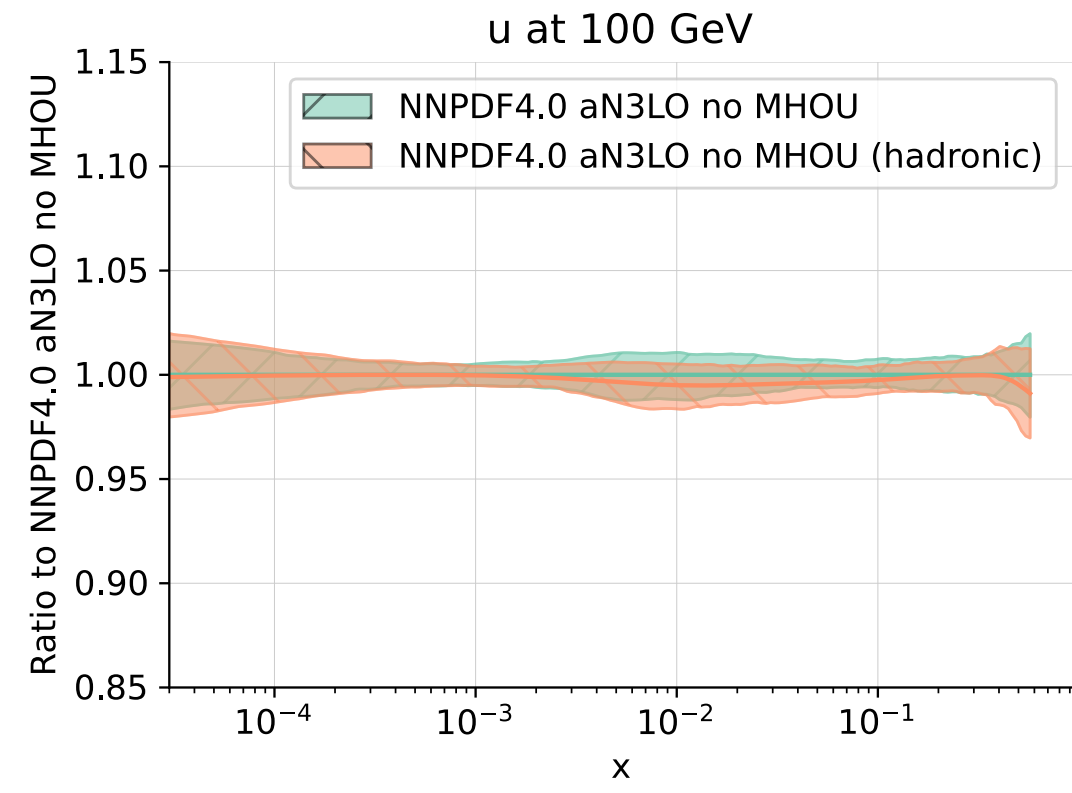


Comparison with MSHT



- Differences similar to those observed at NNLO
- Biggest differences between charm and gluon

Impact of available N3LO corrections for hadronic data

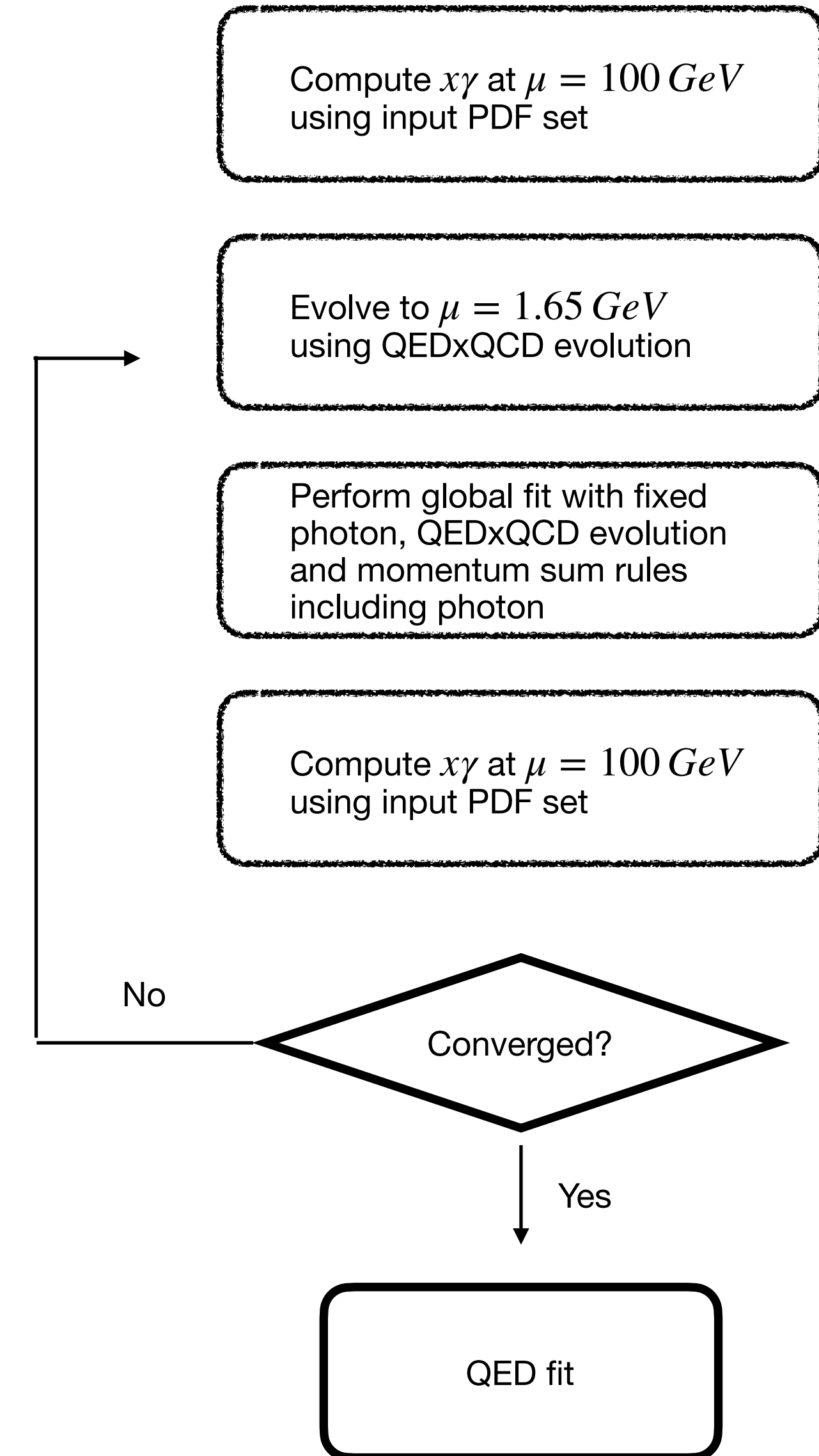
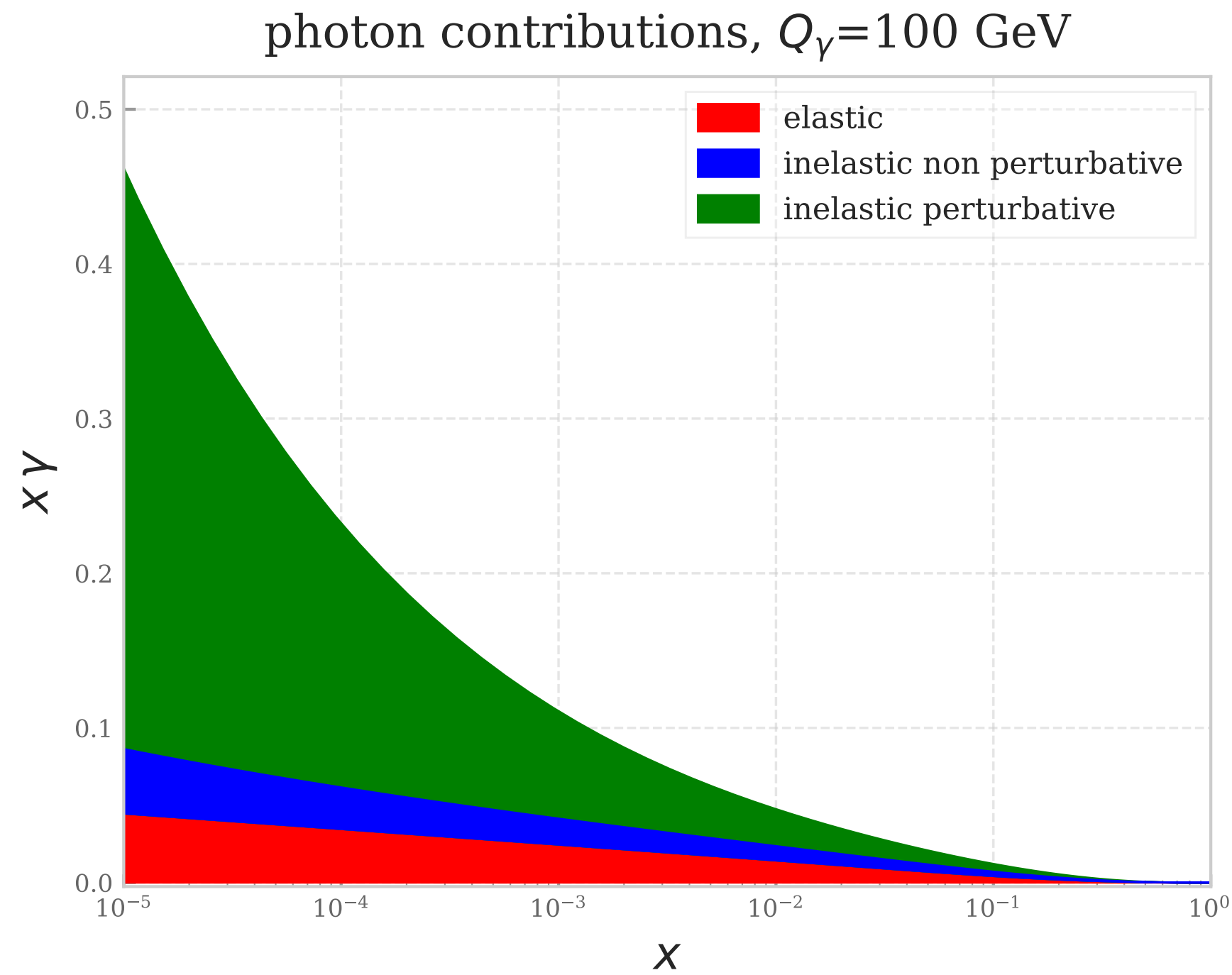


Photons in the proton

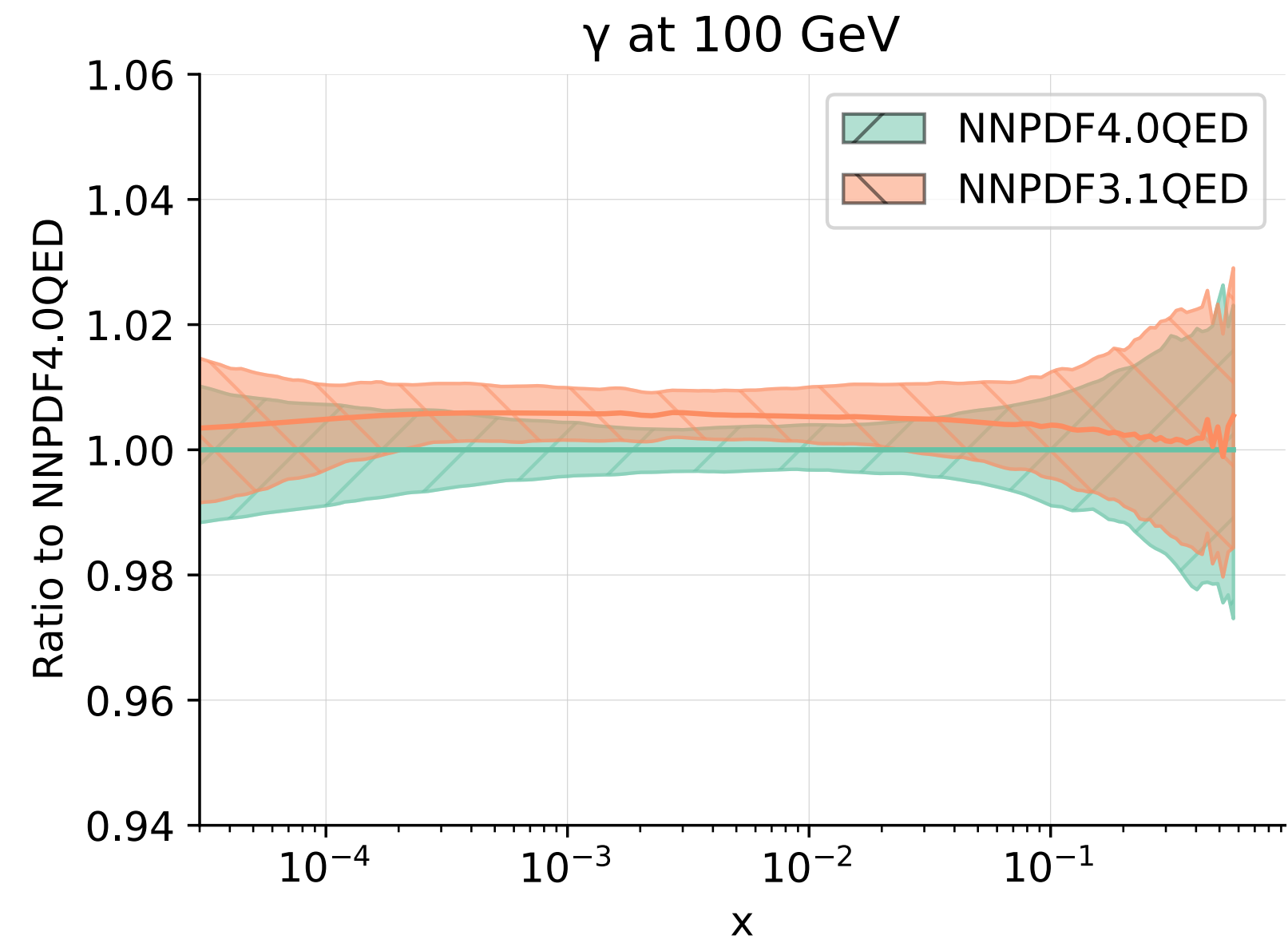
- New NLO and NNLO sets supplementing with a photon PDF the pure QCD PDF set
 - combined QEDxQCD evolution
 - photon PDF determined using the LuxQED formalism

$$x\gamma(x, \mu^2) = \frac{2}{a_{em}(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{m_p^2 x^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} a_{em}^2(Q^2) \left[\left(zP_{\gamma q(z)} + \frac{2x^2 m_p^2}{Q^2} \right) F_2 - z^2 F_L \right] - a_{em}^2(\mu^2) z^2 F_2 \right\}$$

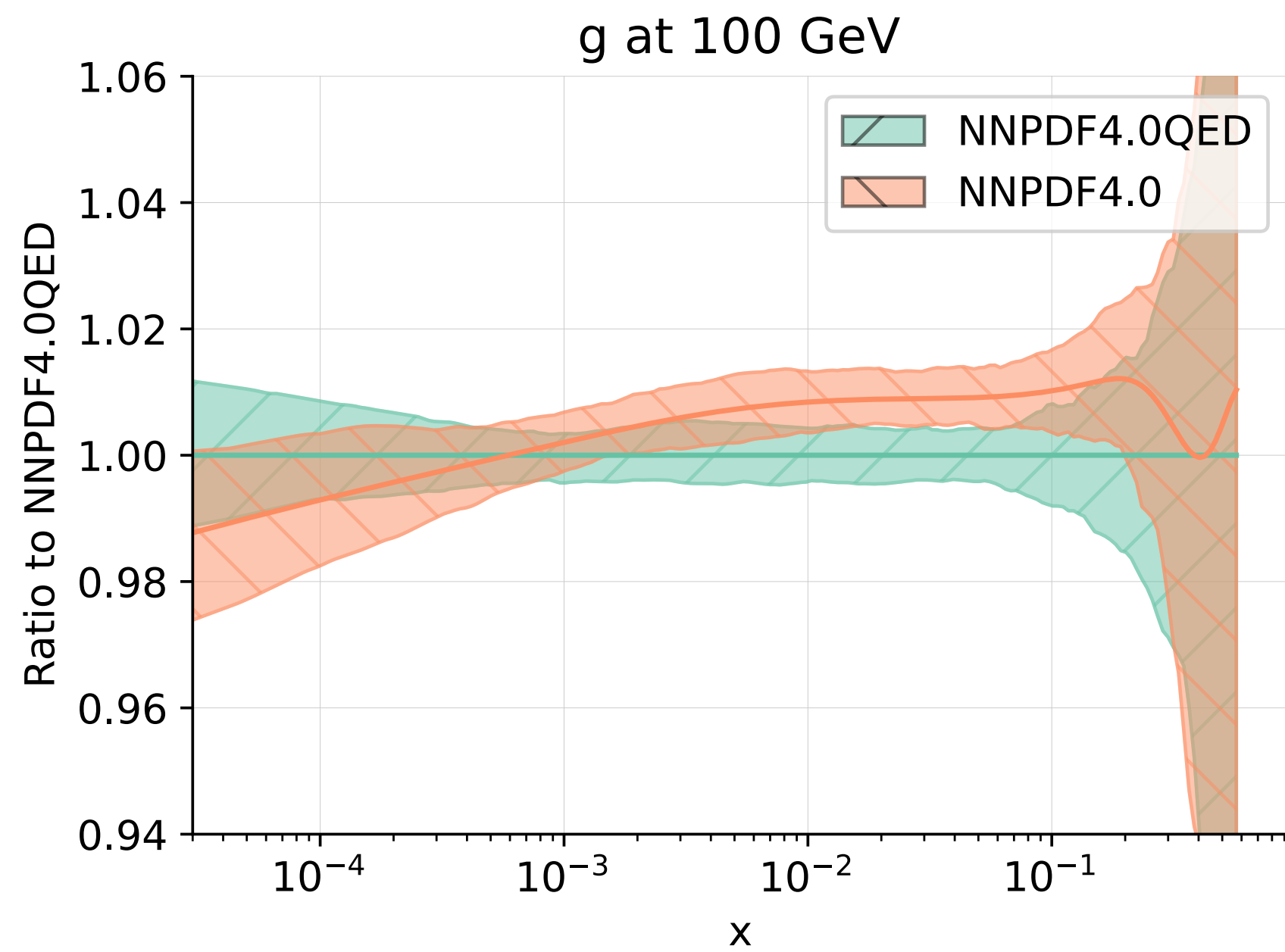
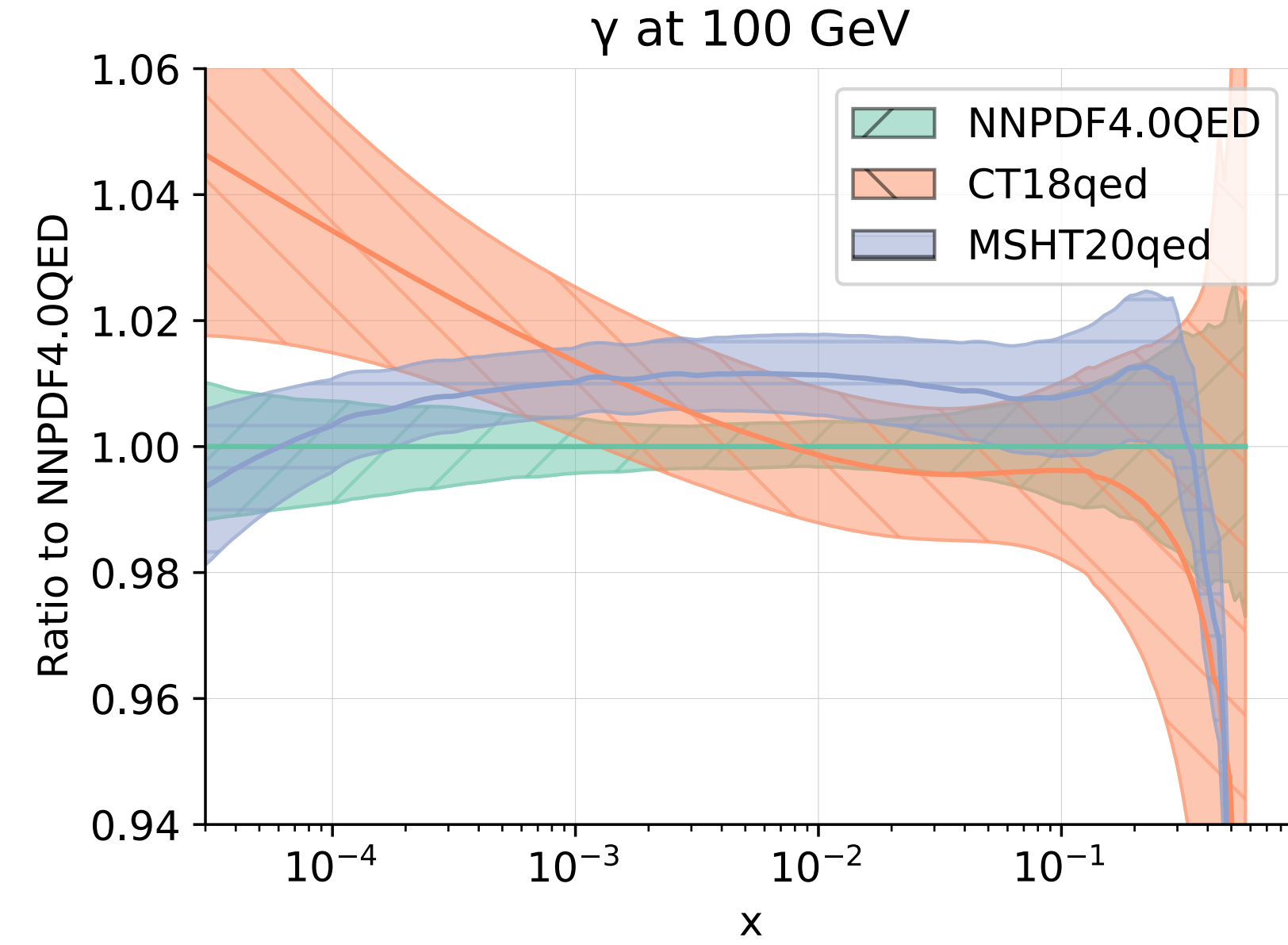
- pure QCD for theory predictions. In particular no photon induced contributions (future work)



Good agreement with previous NNPDF determination...



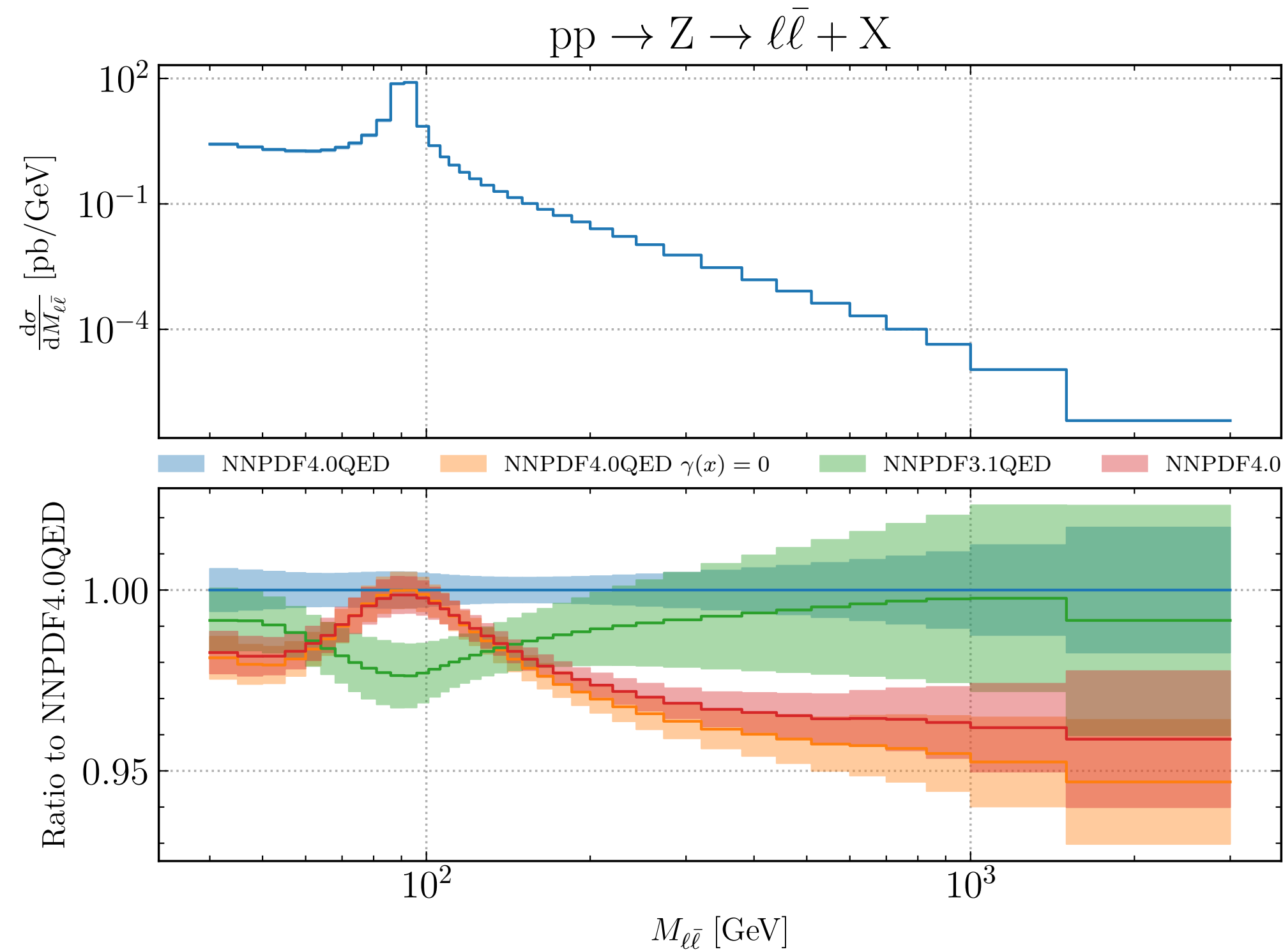
...and with determinations by other groups



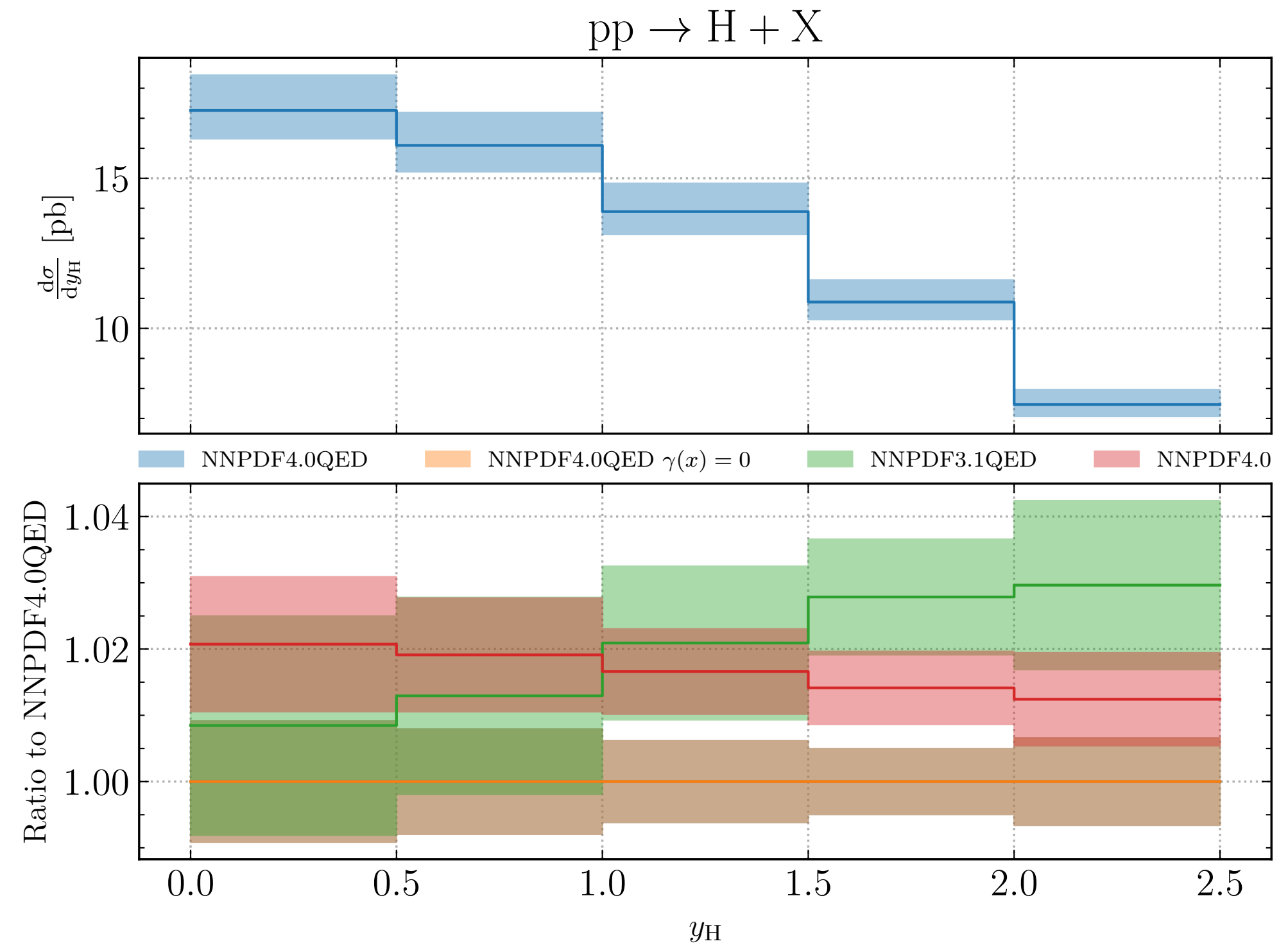
The impact of the photon on the other PDFs is very moderate. Largest effects are seen in the gluon (suppressed according to momentum sum rule)

Implications for LHC phenomenology (only NLO accuracy)

NC DY: Enhancement of cross section due to photon-induced contributions (up to $\sim 5\%$)



Higgs production in gluon fusion: Suppression due to suppression of gluon PDF (up to $\sim 2\%$) and unimportance of photon induced contributions



QED corrections important for precision phenomenology at the percent level