QCD+QED resummation effects in RadISH

Luca Buonocore

based on

[LB, L. Rottoli, P. Torrielli, 2404.15112]

2nd Workshop on Tools for High Precision LHC Simulations

Castel Ringberg - Tegernsee - 8-11 May 2024

Dilepton Drell-Yan (DY): the LHC standard candle

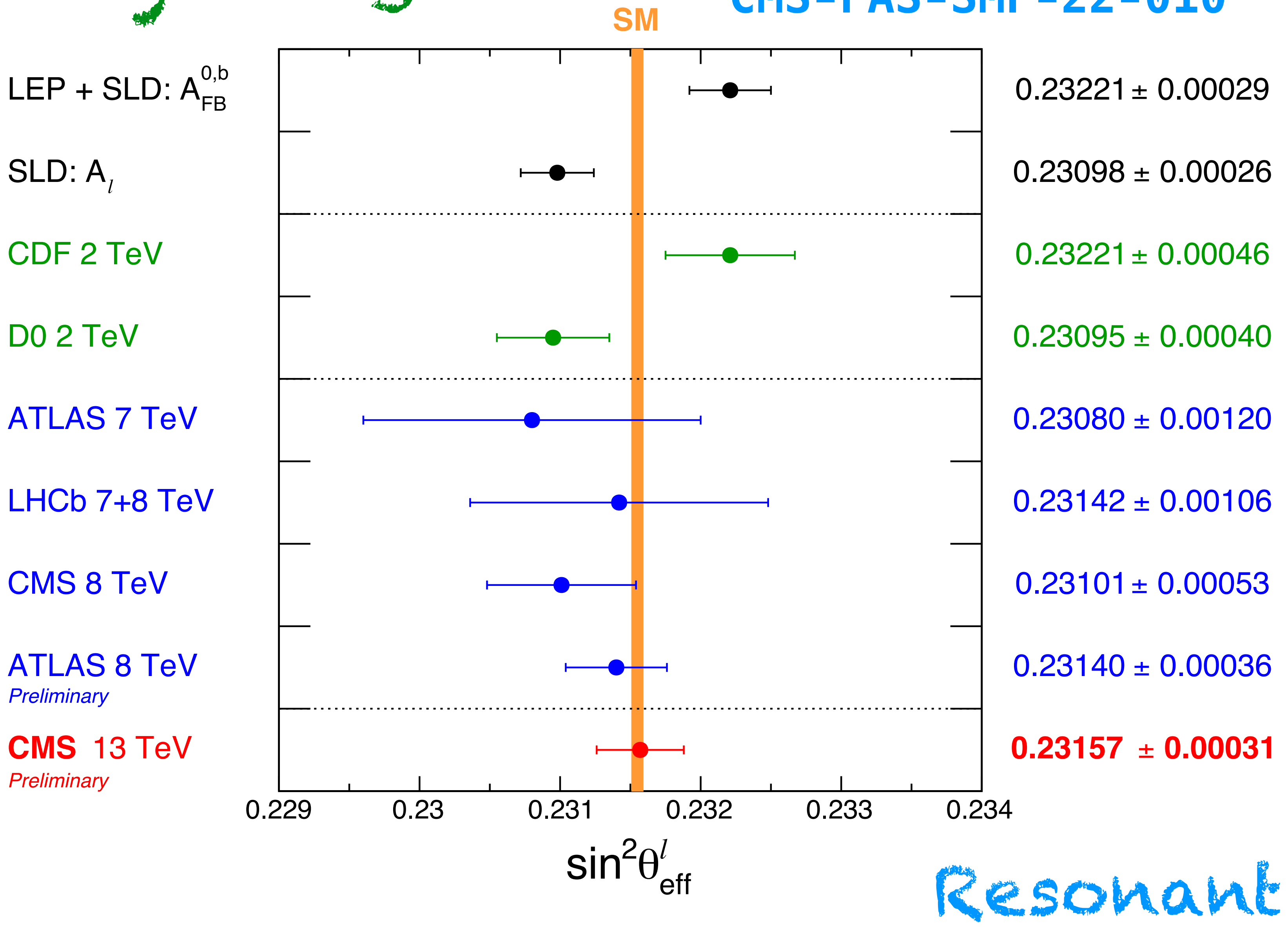
- Measurements of EW precision observables at LHC are becoming competitive with LEP/SLD results
- Control of higher-order radiative corrections crucial for parameter extraction from data
- Sensitivity of precision tests of SM consistency to NP

$$\delta\mathcal{O} \sim Q^2/\Lambda_{NP}^2$$

$$\%\ \text{accuracy at EW} \Longrightarrow \text{scale } \Lambda_{NP} \sim \text{TeV}$$

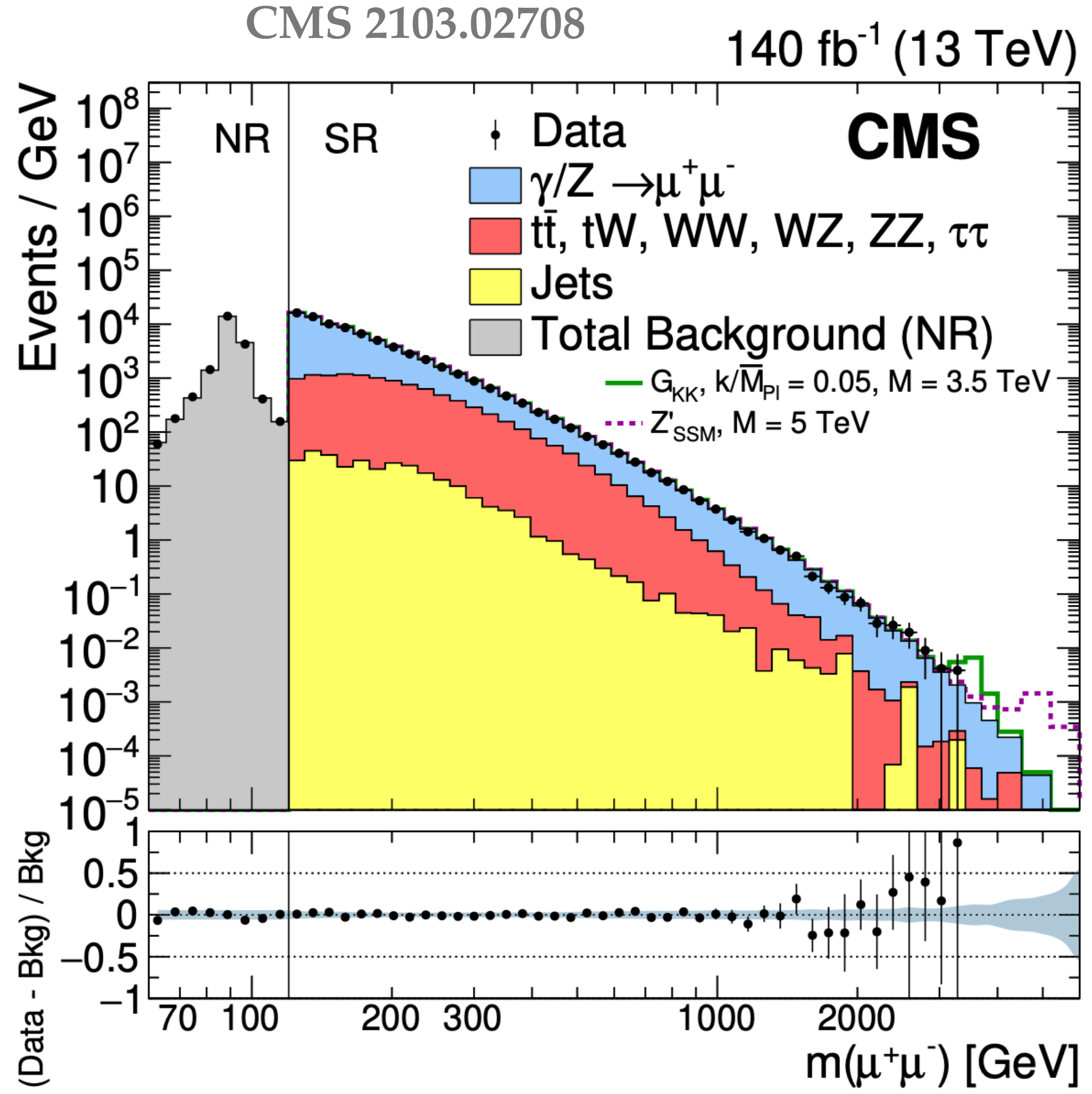
See talk by Long

CMS-PAS-SMP-22-010



Resonant region

Off-shell region



- Measurement of the dilepton invariant mass spectrum **expected at $\mathcal{O}(1\%)$ at $m_{\ell\ell} \sim 1$ TeV**
- Requires control of the SM prediction at the level in the TeV

State-of-art predictions for (NC) Drell-Yan

→ very accurate SM predictions!

$$\sigma = \int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) \hat{\sigma}_{ab}(\hat{s}, \mu_R, \mu_F) + \mathcal{O}(\Lambda/Q)$$

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0,0)}_{ab} + \hat{\sigma}^{(1,0)}_{ab} + \hat{\sigma}^{(2,0)}_{ab} + \hat{\sigma}^{(3,0)}_{ab} + \dots$$
$$+ \hat{\sigma}^{(0,1)}_{ab} + \dots$$
$$+ \hat{\sigma}^{(1,1)}_{ab} + \dots$$

☑ QCD corrections dominant effects

- **NNLO differential cross sections**
  [Anastasiou, Dixon, Melnikov, Petriello (2003)], [Melnikov, Petriello (2006)] [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)] [Catani, Ferrera, Grazzini (2010)]
- **N^3LO inclusive cross sections and di-lepton rapidity distribution**
  [Duhr, Dulat, Mistlberger (2020)] [Chen, Gehrmann, Glover, Huss, Yang, and Zhu (2021)] [Duhr, Mistlberger (2021)]
- **N^3LO fiducial cross sections and distributions**
  [Camarda, Cieri, Ferrera (2021)], [Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli (2022)]

☑ **NLO EW** corrections

- known since long
  [S. Dittmaier and M. Kramer (2002)], [Baur,Wackeroth (2004)], [Baur, Brein, Hollik, Schappacher, Wackeroth (2002)]
- nowadays **automatised** in different available generators
  [Les Houches 2017, 1803.07977]

State-of-art predictions for Drell-Yan: mixed QCD-EW

Theoretical developments

- progress on two-loop master integrals
  [Bonciani, Di Vita, Mastrolia, Schubert (2016)], [Heller, von Manteuffel, Schabinger (2019)], [Hasan, Schubert (2020)]
- renormalization
  [Dittmaier, Schmidt, Schwarz (2020)]
- 2-loop amplitudes for $2 \rightarrow 2$ neutral current DY for massless leptons
  [Heller, von Manteuffel, Schabinger, Spiesberger (2020)]
- 2-loop amplitudes for $2 \rightarrow 2$ neutral current DY (retaining logarithms of the lepton mass)
  [Armadillo, Bonciani, Devoto, Rana, Vicini (2022)]
- 2-loop amplitudes for $2 \rightarrow 2$ charged current DY (retaining logarithms of the lepton mass)
  [Armadillo, Bonciani, Devoto, Rana, Vicini (2024)]

See talk by Buccioni

On-shell Z/W production ($2 \rightarrow 1$ process)

- analytical mixed QCD–QED corrections to the inclusive production of an on- shell Z
  [De Florian, Der, Fabre (2018)]
- fully differential mixed QCD–QED corrections to the production of an on-shell Z
  [Delto, Jaquier, Melnikov, Röntsch (2019)]
- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections
  [Bonciani, Buccioni, Rana, Vicini (2020)]
- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections
  [F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch (2020)], [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)]

State-of-art predictions for Drell-Yan: mixed QCD-EW

Beyond on-shell computations

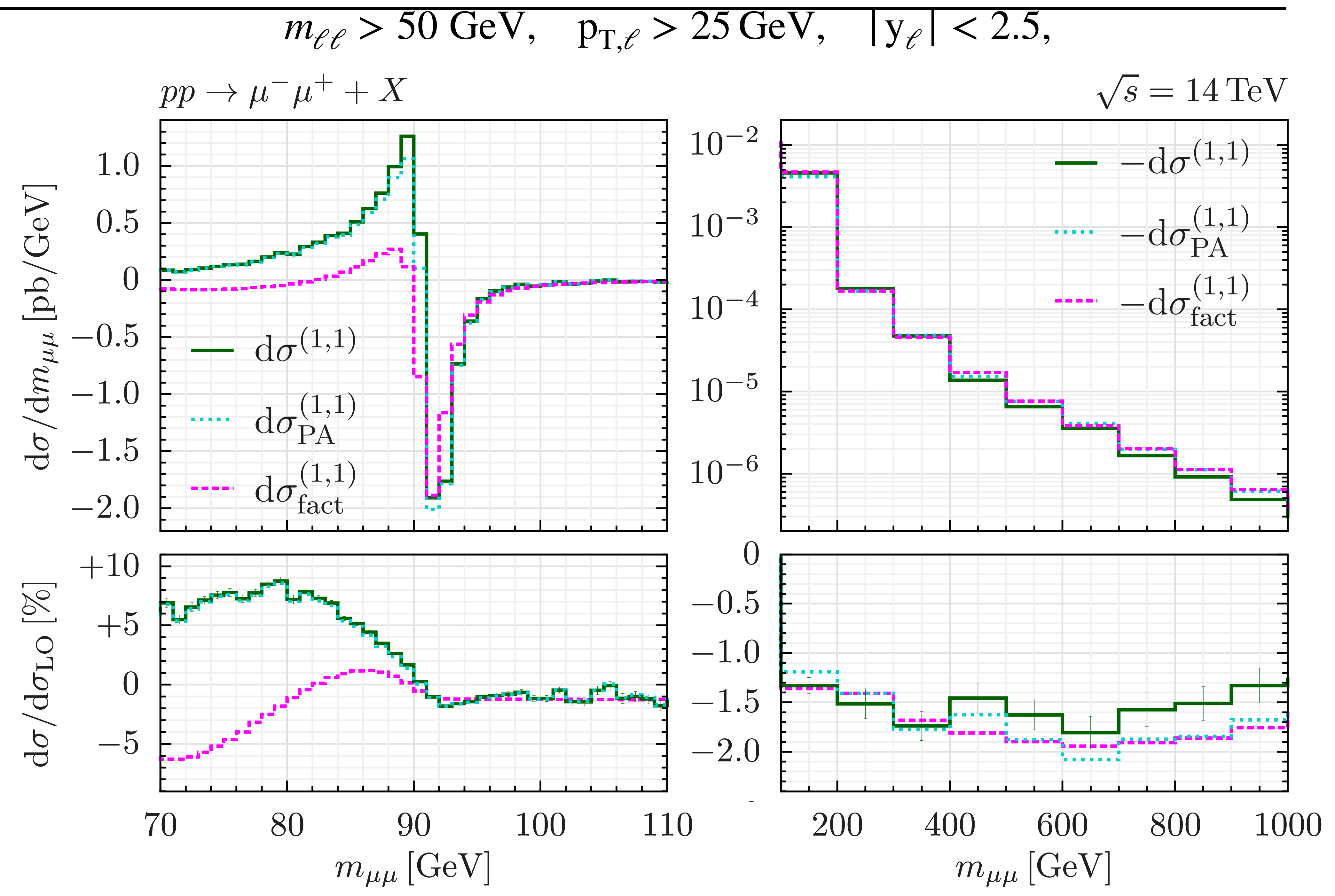
- dominant Mixed QCD-EW corrections in Pole Approximation for neutral- and charged- DY processes
  [Dittmaier, Huss, and Schwinn (2014,2015)]
- approximate corrections available in parton showers based on a factorised approach
  [Balossini et al (2010)], [Bernaciak,Wackeroth (2012)], [Barze' et al (2012,2013], [Calame et al (2017)]
- neutrino-pair production including NNLO QCD-QED corrections
  [Cieri, Der, De Florian, Mazzitelli (2020)]
- complete mixed QCD-EW corrections in Pole Approximation and impact on the Forward-Backward asymmetry
  [Dittmaier, Huss, and Schwarz (2024)]

State-of-art predictions for Drell-Yan

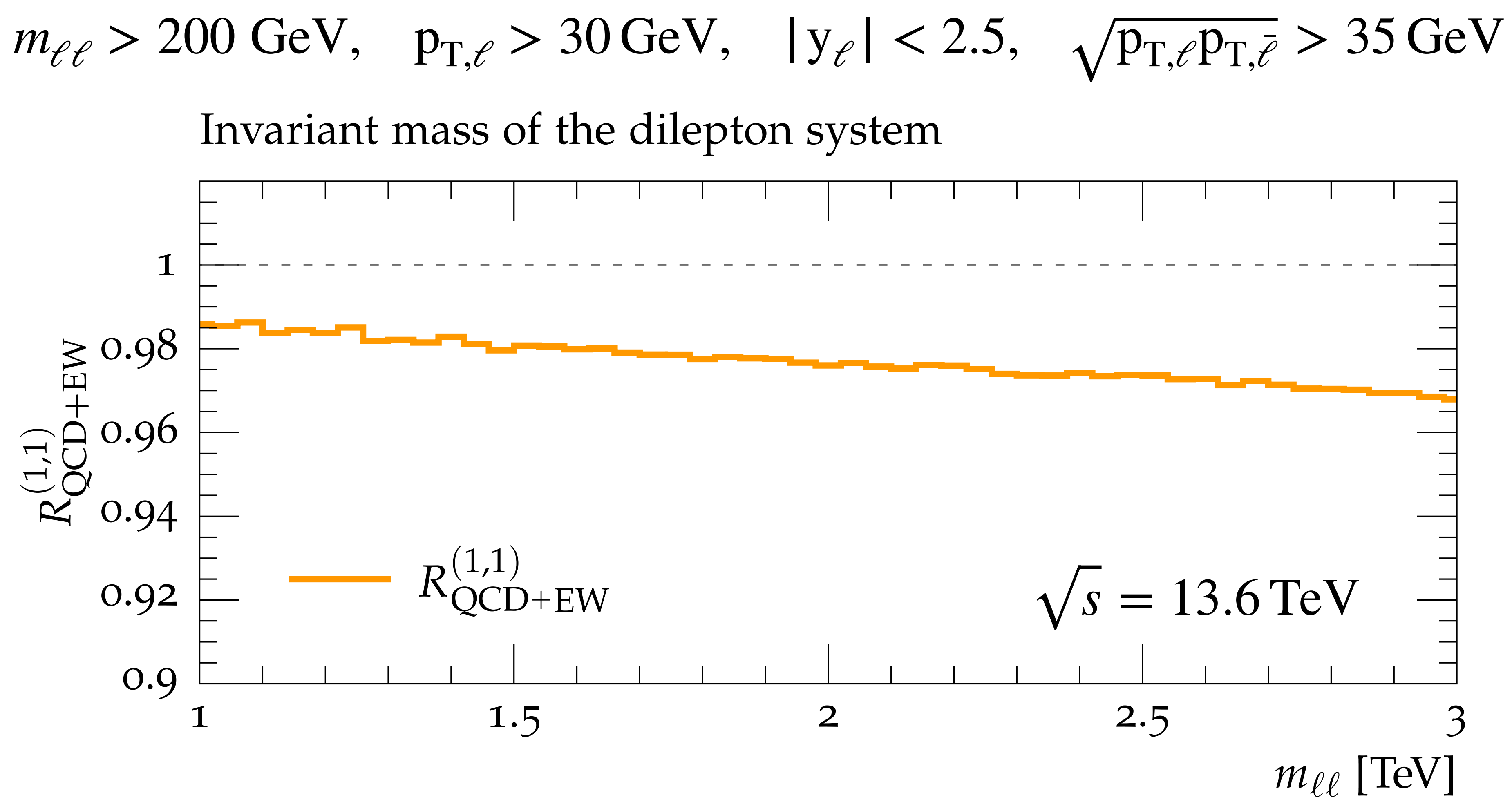
NC current Drell-Yan

Bare muons (massive calculation)
[Bonciani, LB, Grazzini, Kallweitt, Rana, Tramontano, Vicini '21]

Impact at large invariant masses (massless leptons)
[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melinokov, Röntsch, Signorile-Signorile et al '22]



- The Drell-Yan is the cornerstone of the precision physics program at the LHC (m_W, $\sin\theta_W$, α_S extractions)
- **Negligible?**
  mixed QCD-EW parametrically of similar importance as N³LO in QCD
- **Factorized ansatz?**
  is a multiplicative combination of QCD and EW justified?

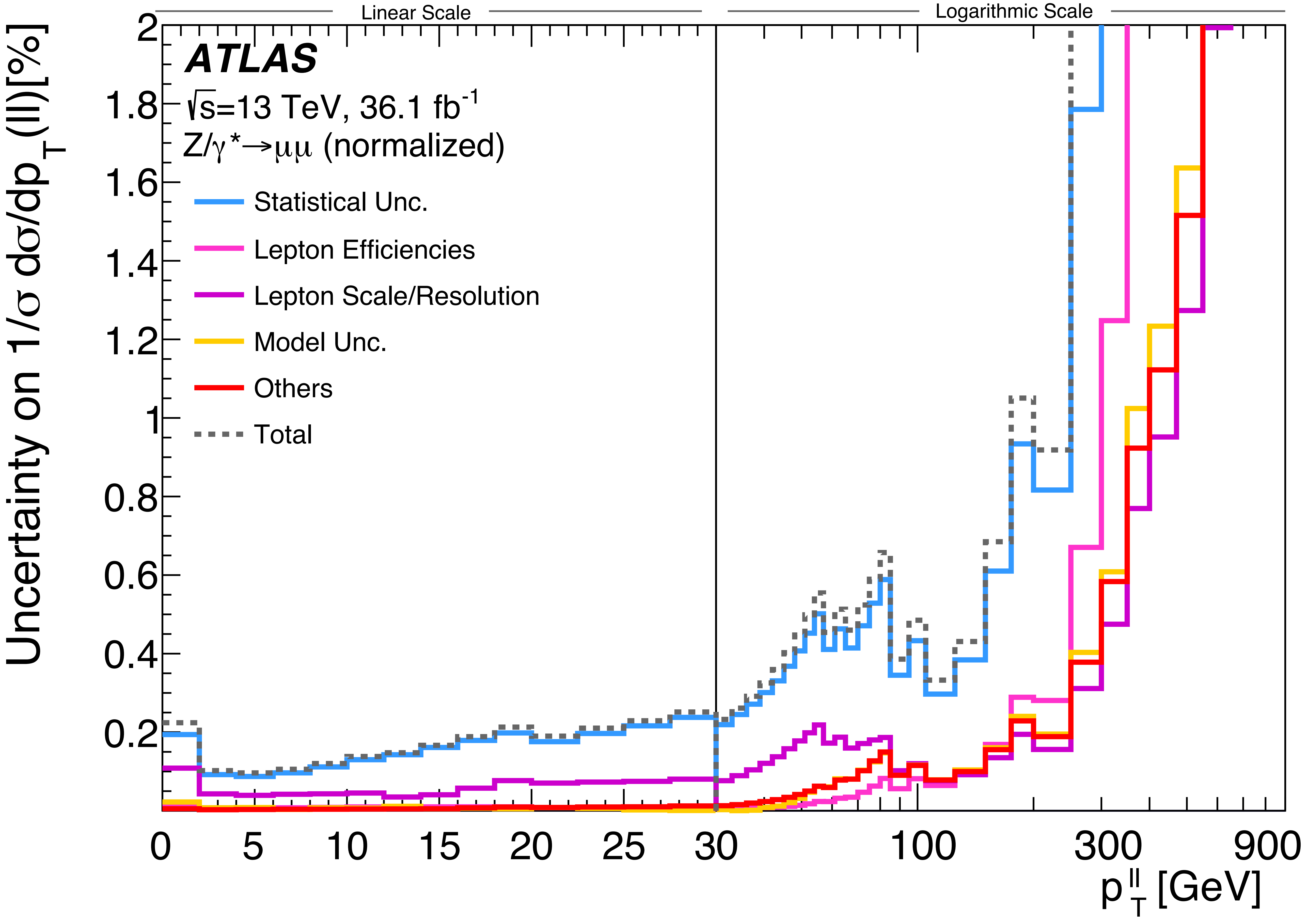


Non-negligible impact at high invariant masses

But well described by the product of QCD and EW (large Sudakov log) corrections

State-of-art predictions for Drell-Yan

Not only fixed order:
(Transverse momentum) resummation very important!

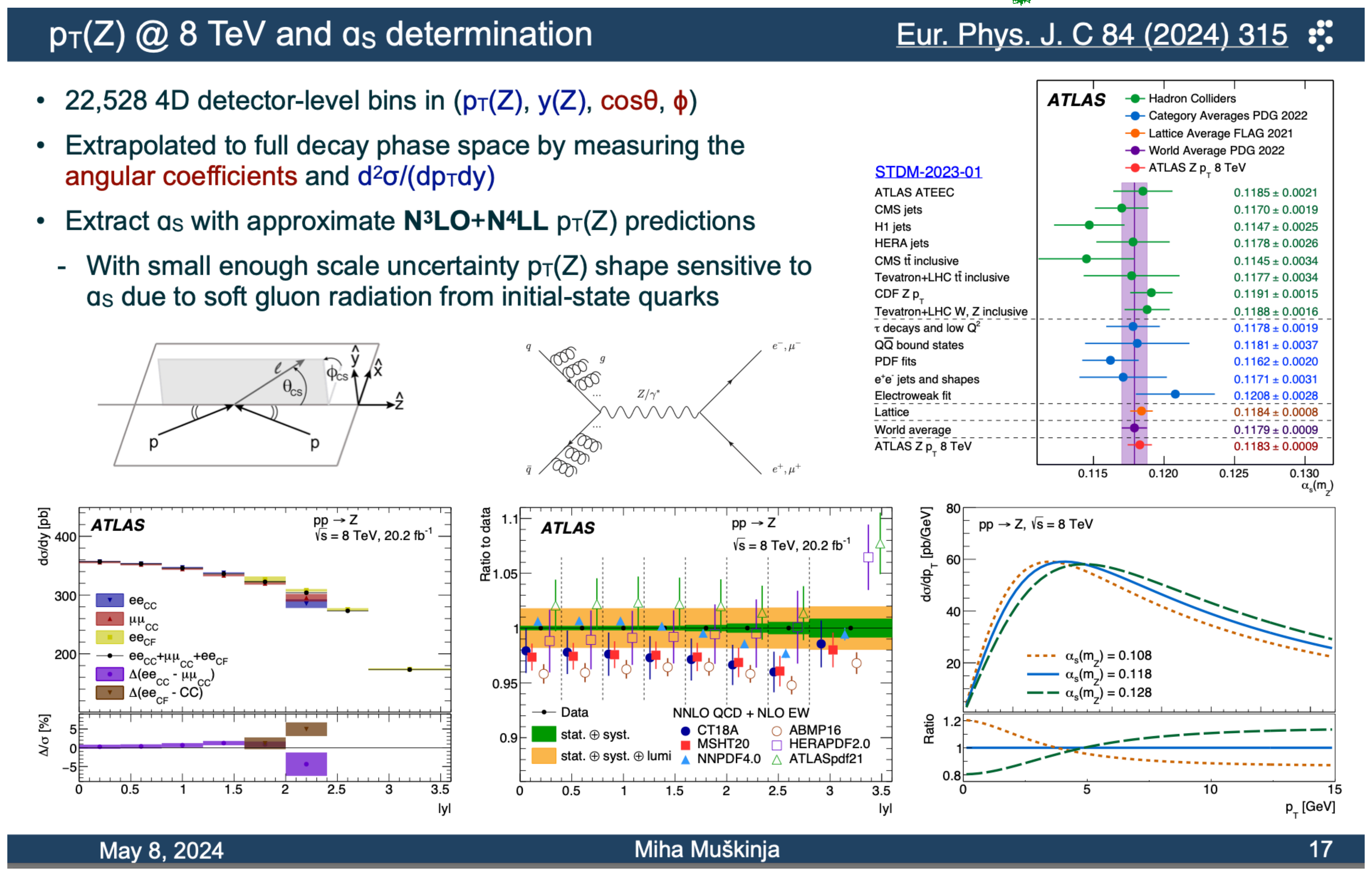


Current state of art: N^3LL' and first approximate results at aN^4LO

[Re, Rottoli, Torrielli (2021)], [Camarda, Cieri, Ferrera (2021)], [Ju, Schönherr (2021)], [Camarda, Cieri, Ferrera (2023)]

Used for comparison with data and **extraction of parameters**

See talk by Muskinja



Motivations and objective

CAVEAT: comparisons usually made at the level of pure QCD models considering "Born" lepton
Large FSR QED effects are subtracted by experimentalists relying on Monte Carlo modeling (PHOTOS)
[Barberio, van Eijk ,Was '91][Golonka, Was, '06]

- From a theoretical point of view, the definition of "Born" leptons is not ideal
- It provides a good description of the main QED effects but makes less transparent the impact of full EW effects and the interplay with QCD corrections (underlying assumption of complete factorization)
- How to estimate uncertainties?
- Unfolded data for bare/dressed letptons ?

GOAL: combining higher-order QCD resummation with the resummation of leading EW and mixed QCD-EW effects in a flexible "analytic" resummation tool, including matching to available fixed-order results

Going beyond the on-shell approximation which does not include treatment of final-state leptons

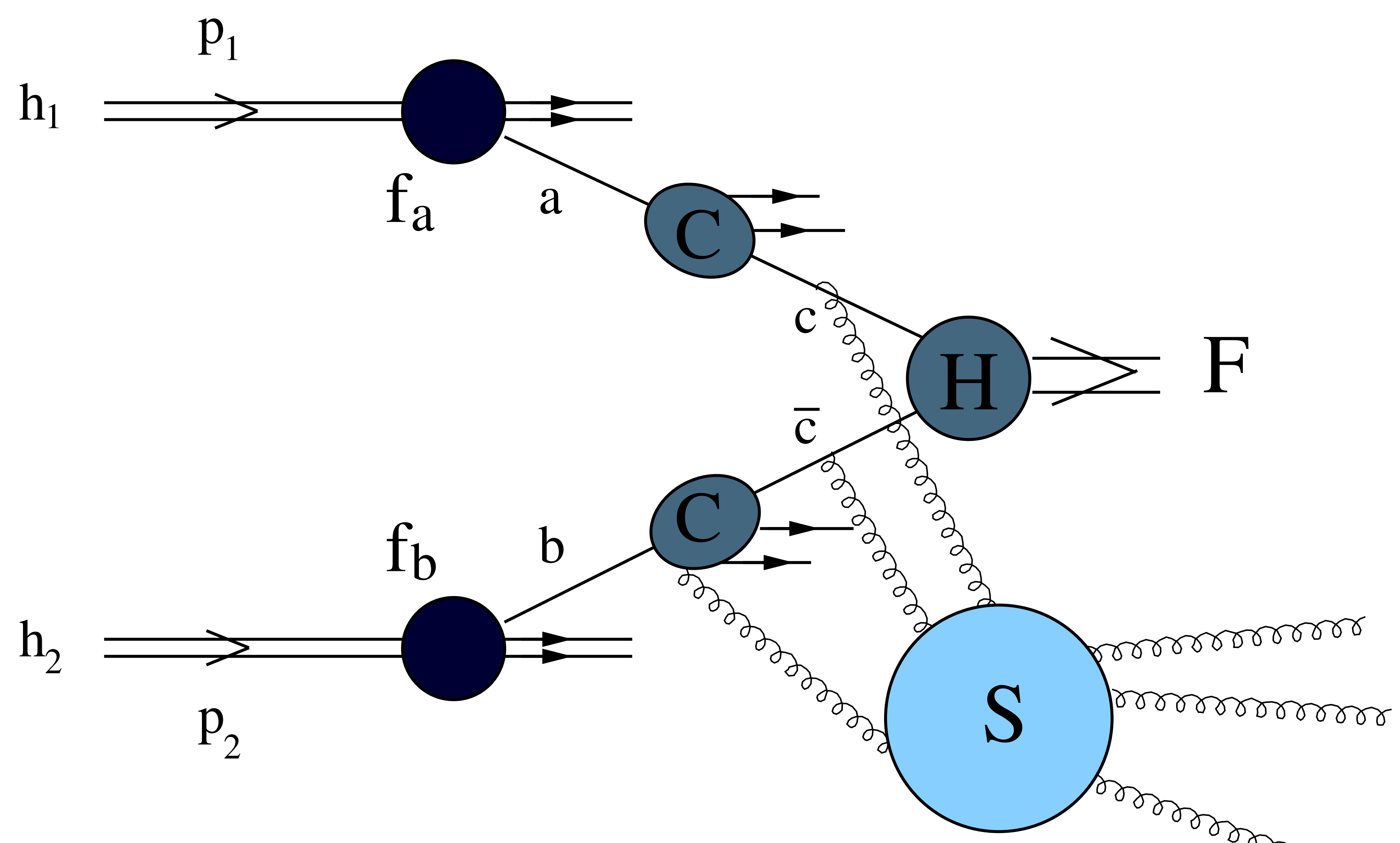
[Cieri, Ferrera, Sborlini, 2018][Autieri, Cieri, Ferrera, Sborlini, 2023]

q_T-resummation (QCD): color-less final state

color-less system F: (Q^2, Y, q_T)

[Catani, de Florian, Grazzini, 2001]

$$\frac{d\sigma^{(sing)}}{dQ^2 dY dq_T d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},F}}{d\Omega}\int_0^\infty db\frac{b}{2}J_0(bq_T)S_c(Q,b)\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}[H^F C_1 C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2)$$



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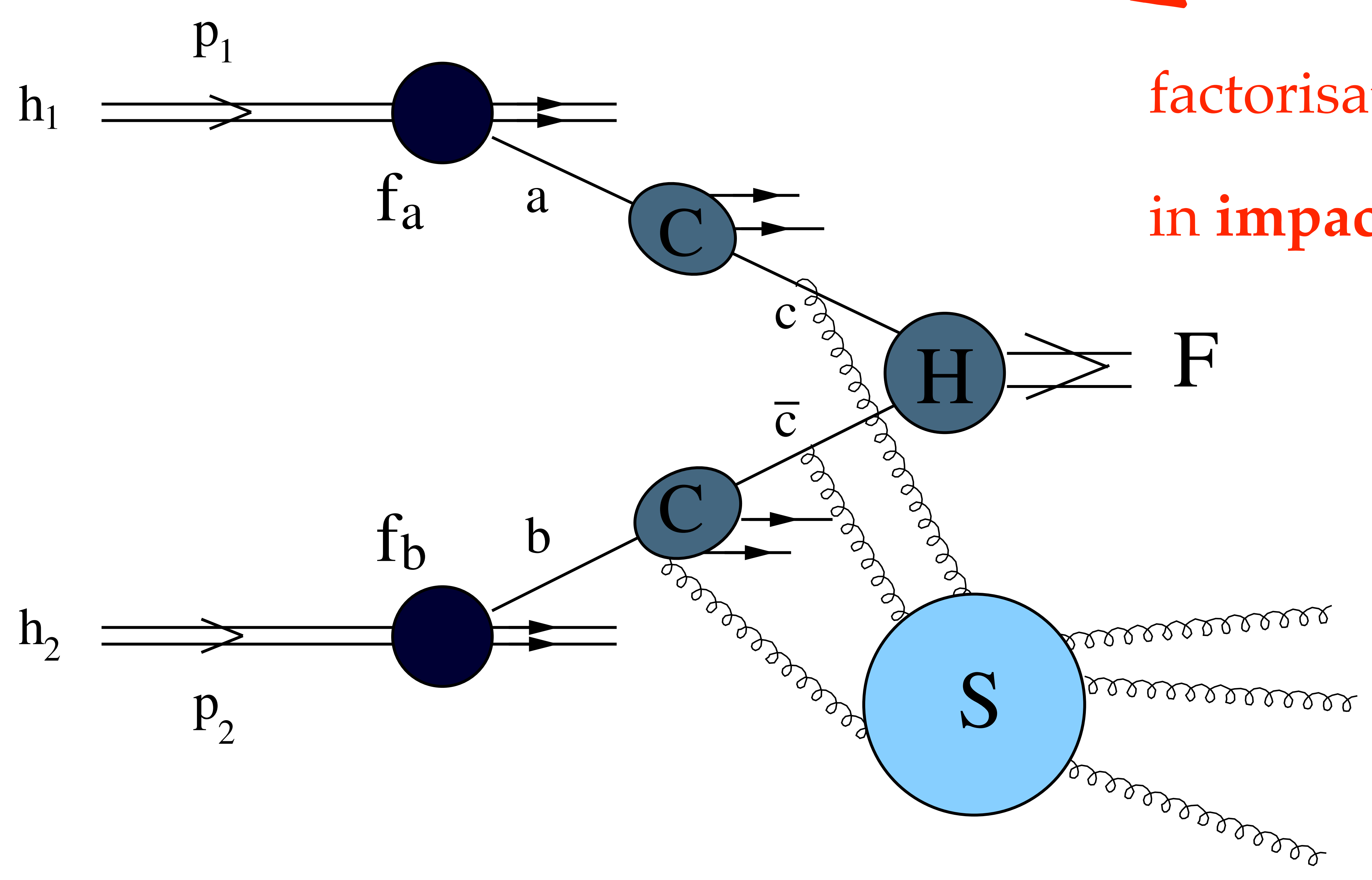

factorisation of the constraint $\delta^2\left(\mathbf{q}_T - \sum_i \mathbf{k}_{T,i}\right)$

in **impact parameter space**

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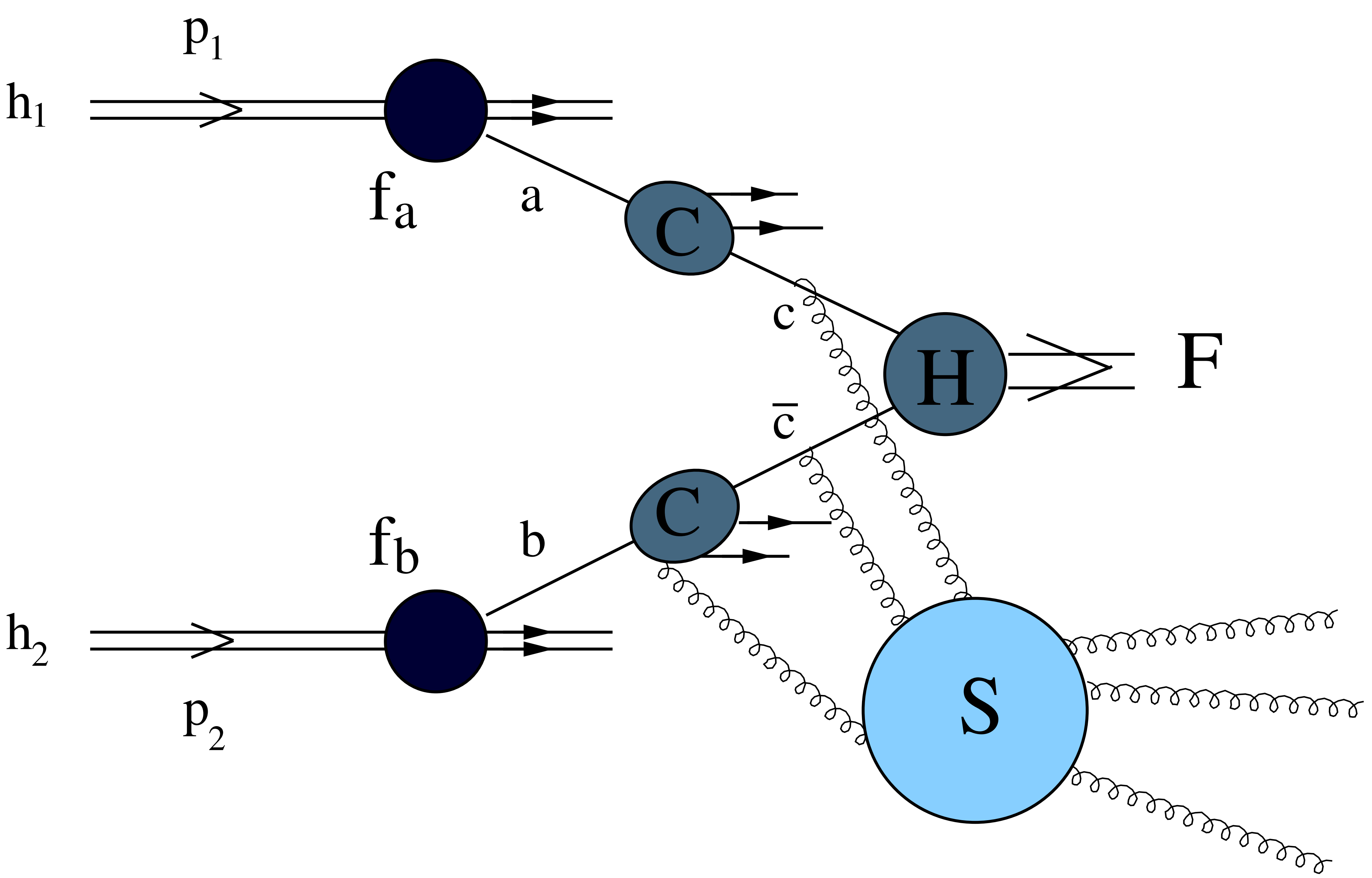

Universal **Sudakov Form Factor**:
exponentiation of soft-collinear emissions

$$S_c(Q,b) = \exp\left[-\int_{b_0^2/b^2}^{Q^2} dq^2 A_c\left(\alpha_S(q^2)\right)\ln\frac{Q^2}{q^2} + B_c\left(\alpha_S(q^2)\right)\right]$$

A_c, B_c admits a perturbative expansion in α_S

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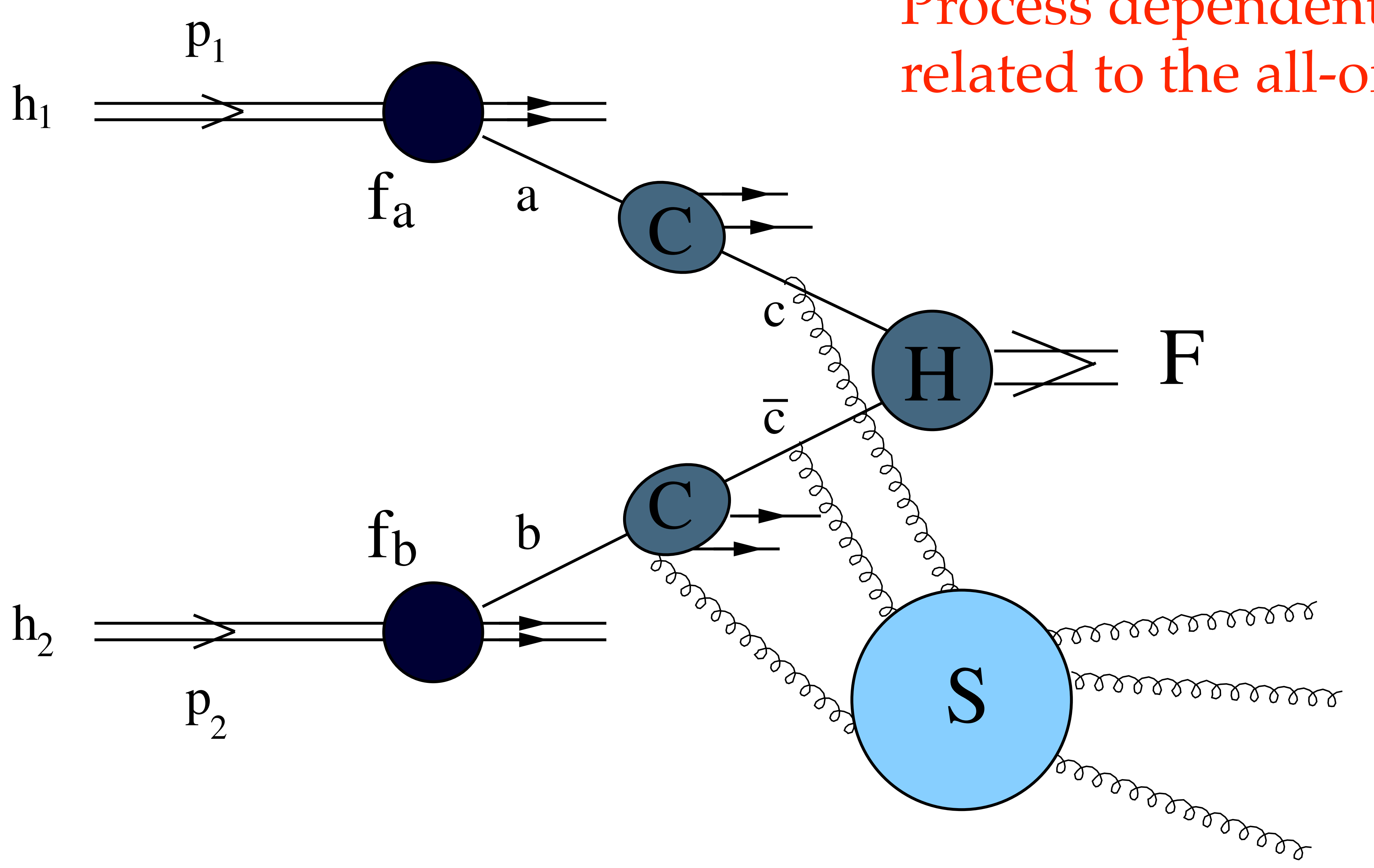
Process dependent **Hard-Virtual function** related to the all-order elastic amplitude

Universal **collinear or beam function**



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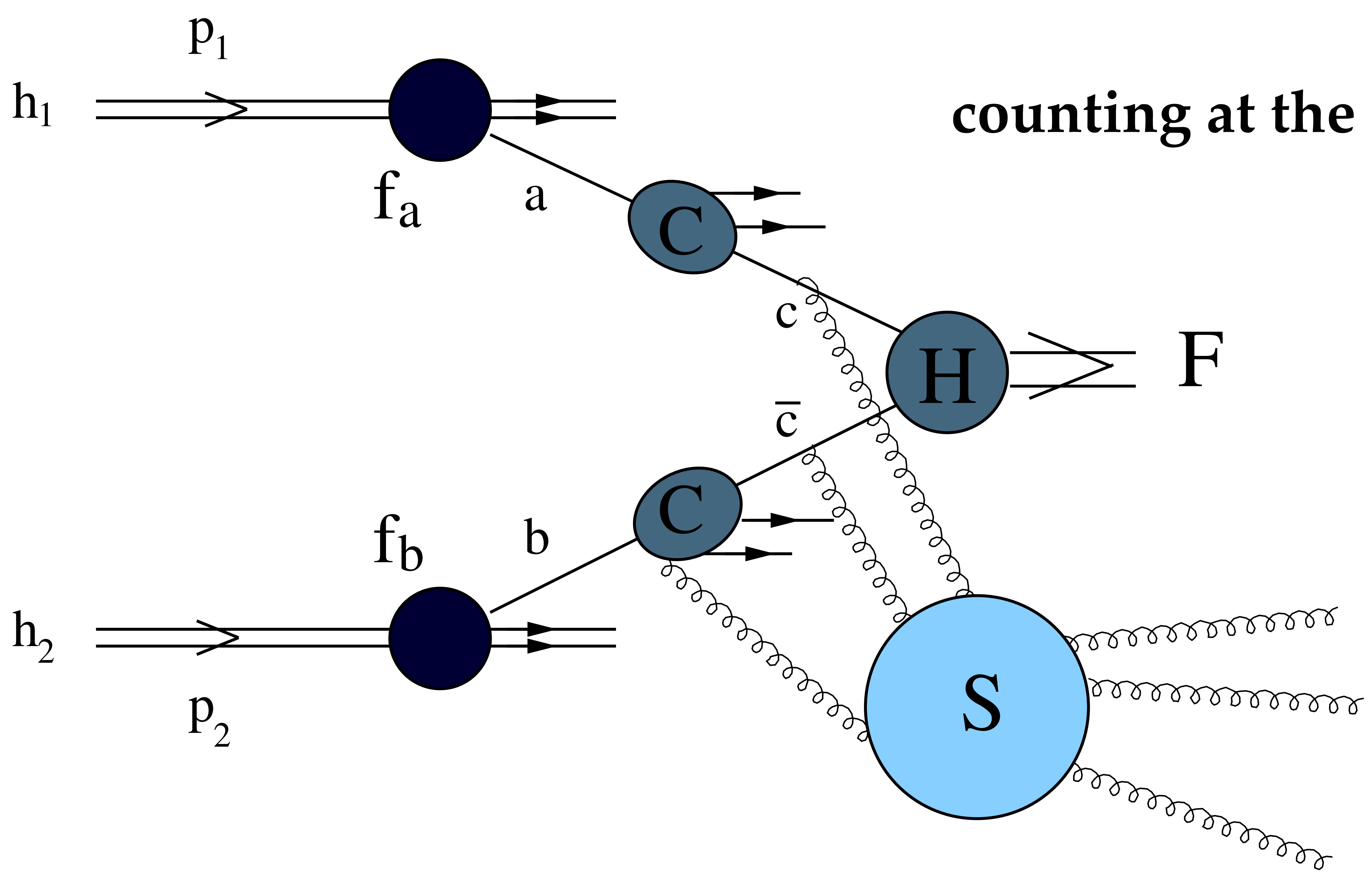
expansion parameter: $\alpha_S(Q) \times \ln\frac{Q^2 b^2}{b_0^2} = a_S L \sim 1$



counting at the level of the exponent $\sim \exp[L g_1 + g_2 + \frac{\alpha_s}{\pi} g_3]$

LL	NLL	NNLL	requires:
$\alpha_S L^2$	$\alpha_S L$		$A_c^{(1)}$
$\alpha_S^2 L^3$	$\alpha_S^2 L^2$	$\alpha_S^2 L$	
⋮	⋮	⋮	
$\alpha_S^k L^{k+1}$	$\alpha_S^k L^k$	$\alpha_S^k L^{k-1}$	
⋮	⋮	⋮	

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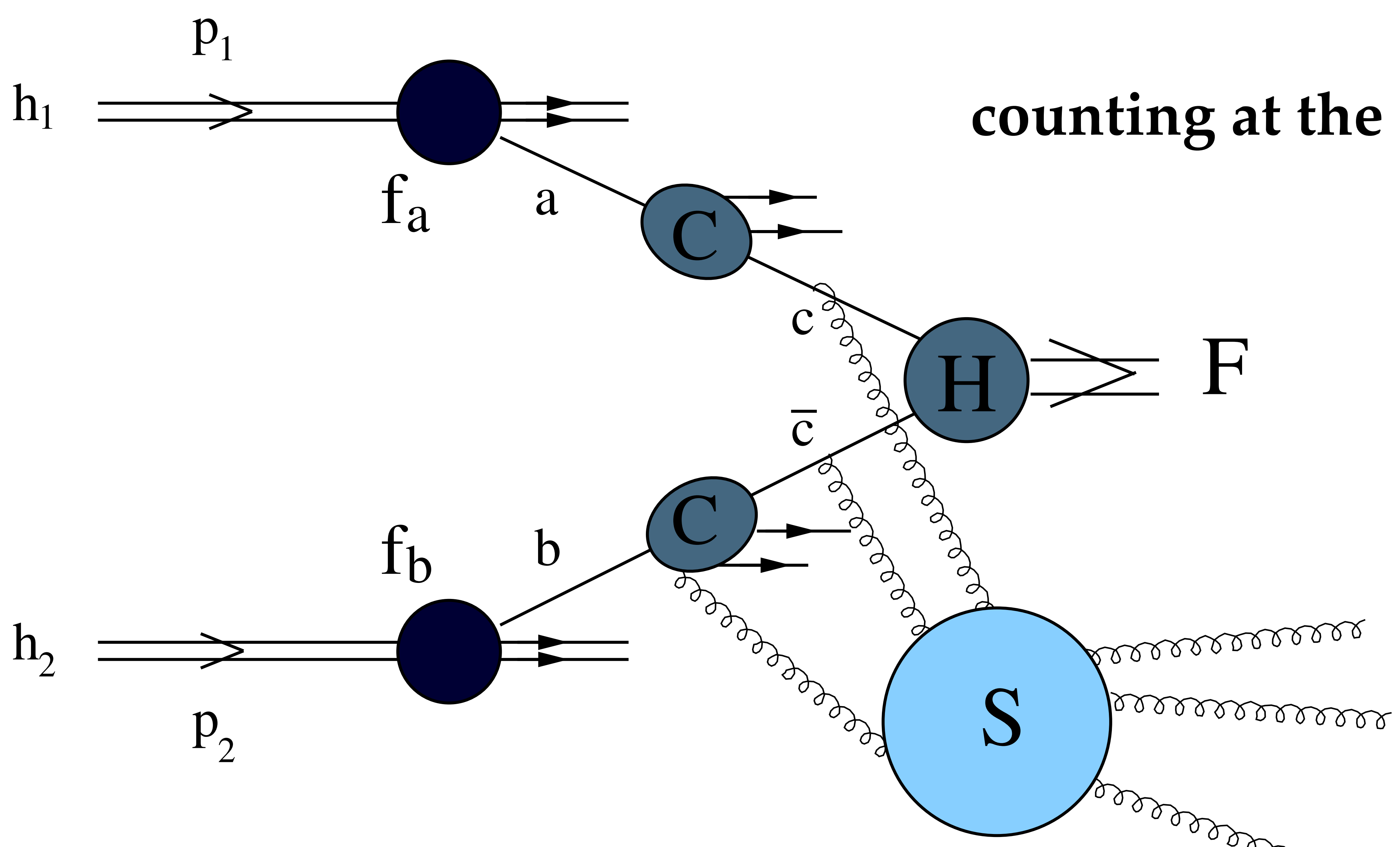

LL	NLL	NNLL
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$\vdots$	$\vdots$	$\vdots$
$\alpha_S^k L^{k+1}$	$\alpha_S^k L^k$	$\alpha_S^k L^{k-1}$
$\vdots$	$\vdots$	$\vdots$

requires:

$A_c^{(1)}, A_c^{(2)}, B_c^{(1)}, C_c^{(1)}, H_F^{(1)}$

(plus **beta function** at 2loop and **collinear anomalous dimensions** at 1loop)

(q_T subtraction @ NLO)

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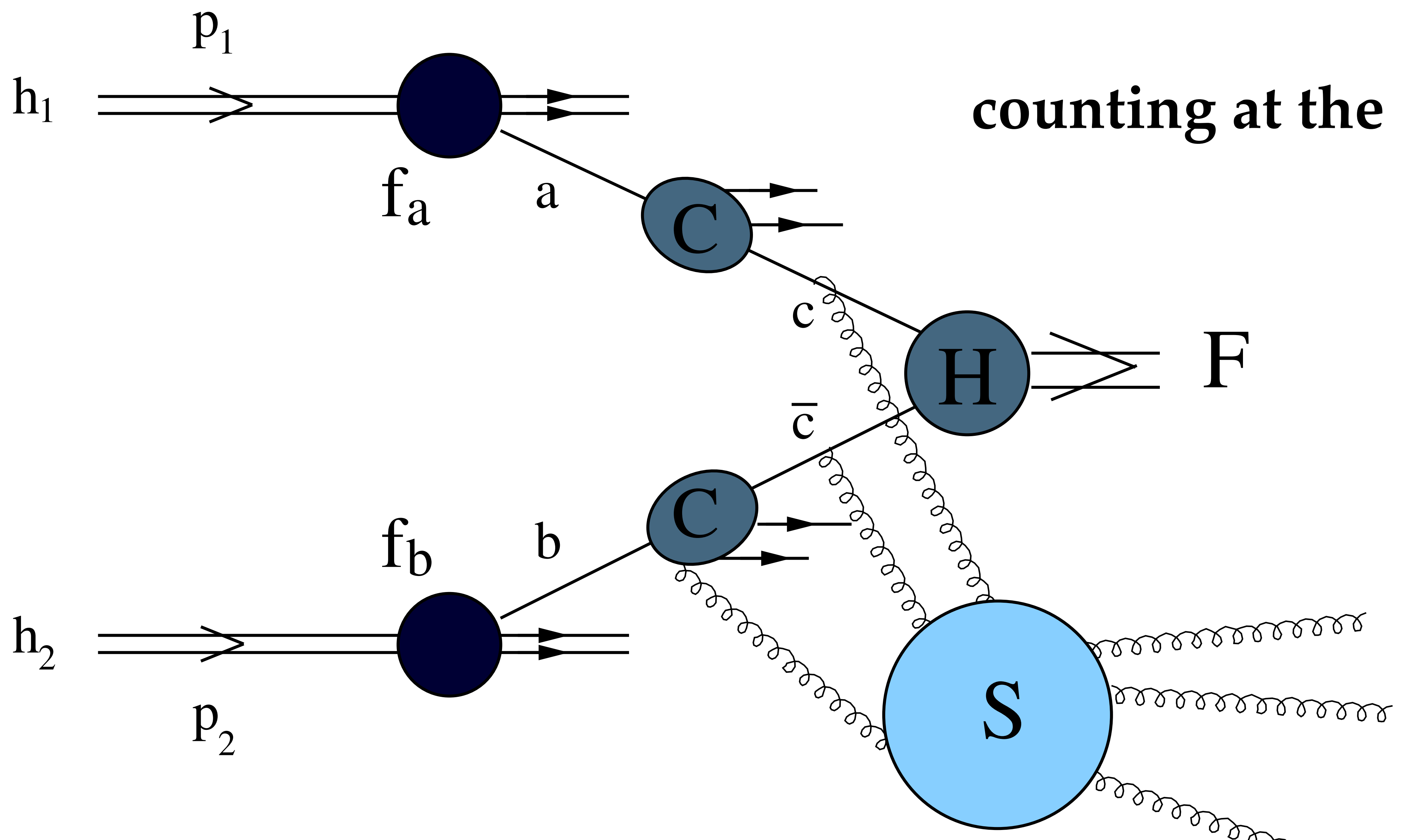

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requires:
$A_c^{(1)}, A_c^{(2)}, B_c^{(1)}, C_c^{(1)}, H_F^{(1)},$
$A_c^{(3)}, B_c^{(2)}, C_c^{(2)}, H_F^{(2)}$
plus **beta function** at 2loop and **collinear anomalous dimensions** at 2loop)

(q_T subtraction @ NNLO)

q_T-resummation QCD-QED(EW)

Definition of the observable

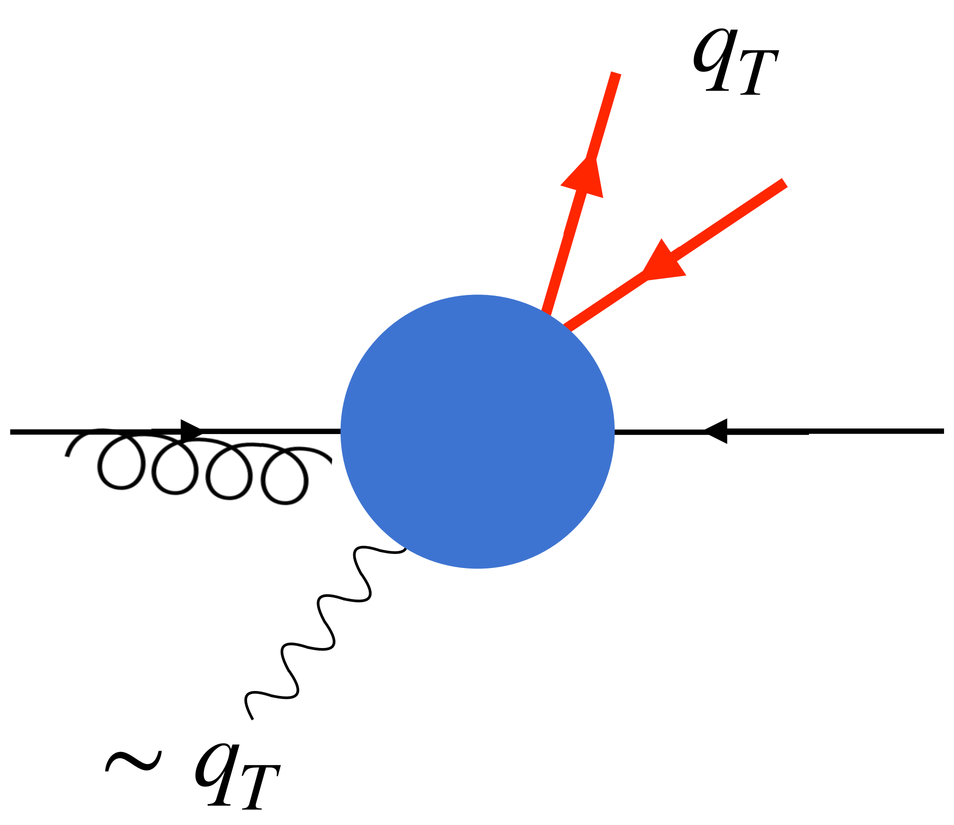
The transverse momentum of the final-state system controls the radiation emitted from initial-state partons

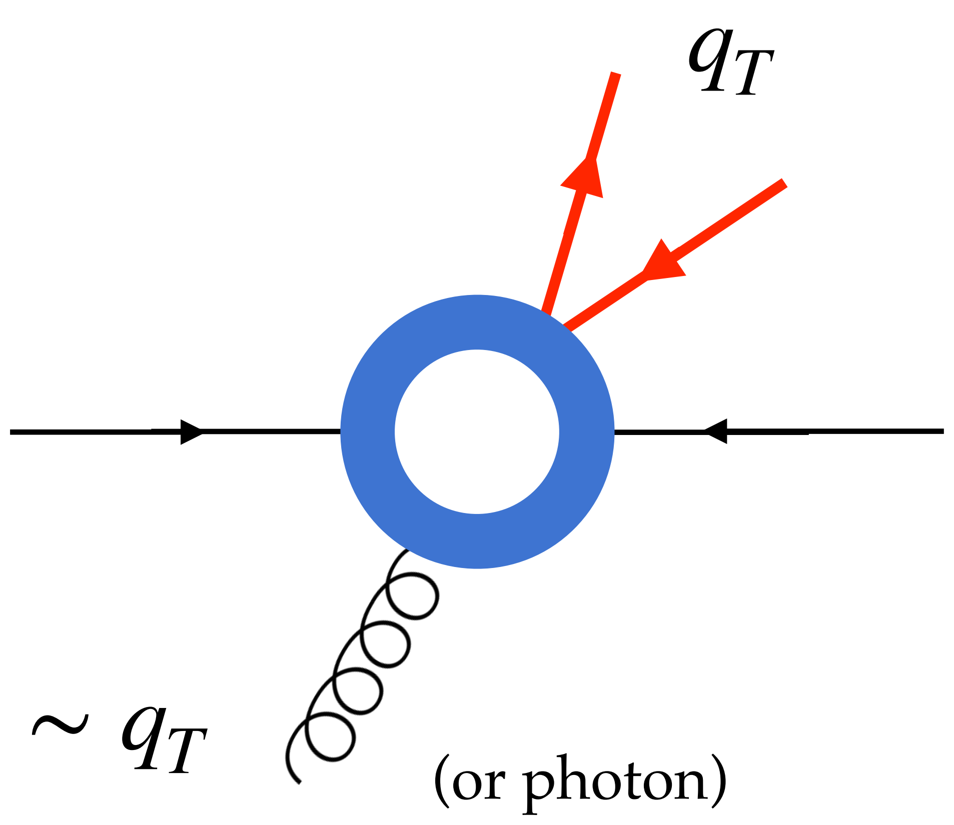
In the presence of **massless radiators** in the final state at LO, a different observable must be used, like N-jettiness (or k_T-ness)

Example: mixed QCDxEW corrections to Drell-Yan dilepton production $p + p \to \ell + \bar{\ell} + X$

Initial-state radiation

For $q_T > 0$ **one emission is always resolved**





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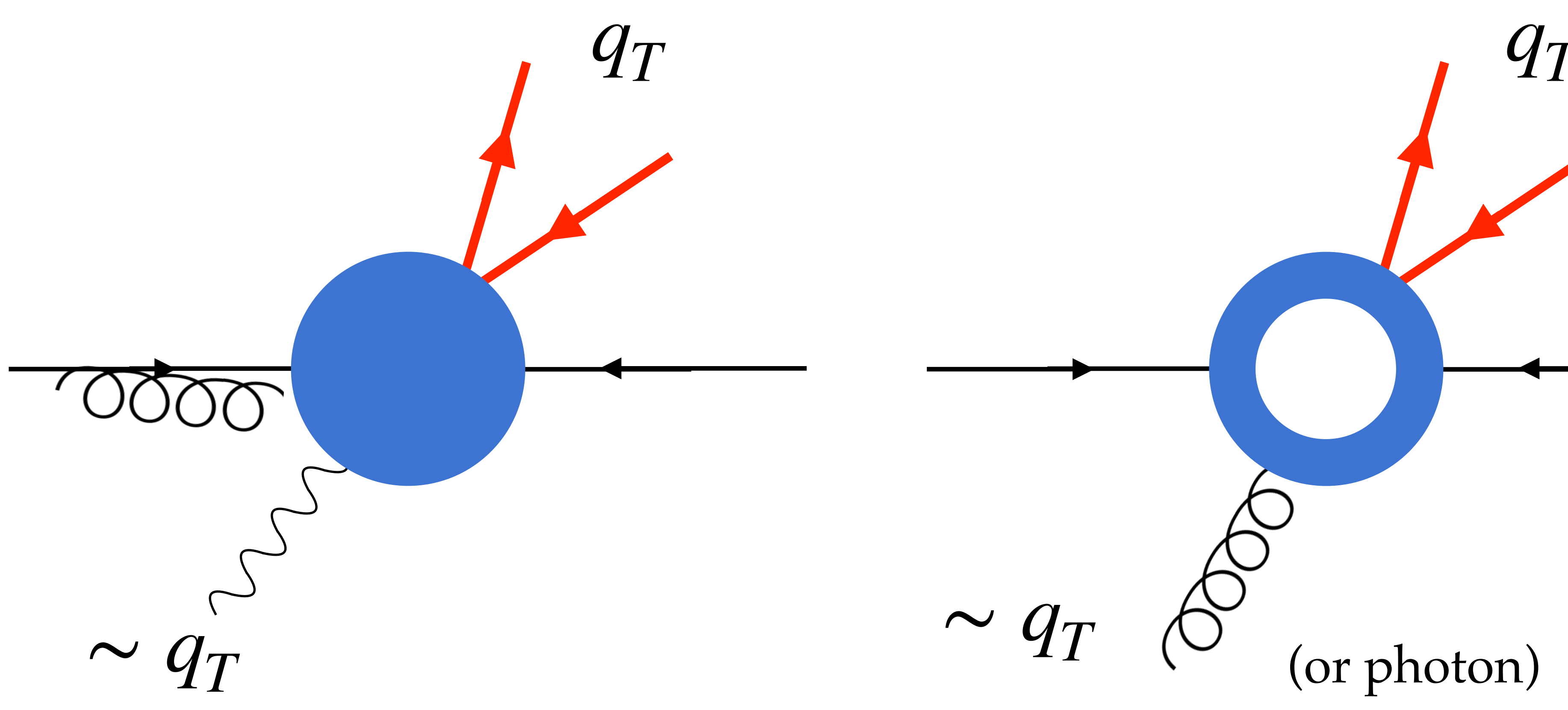
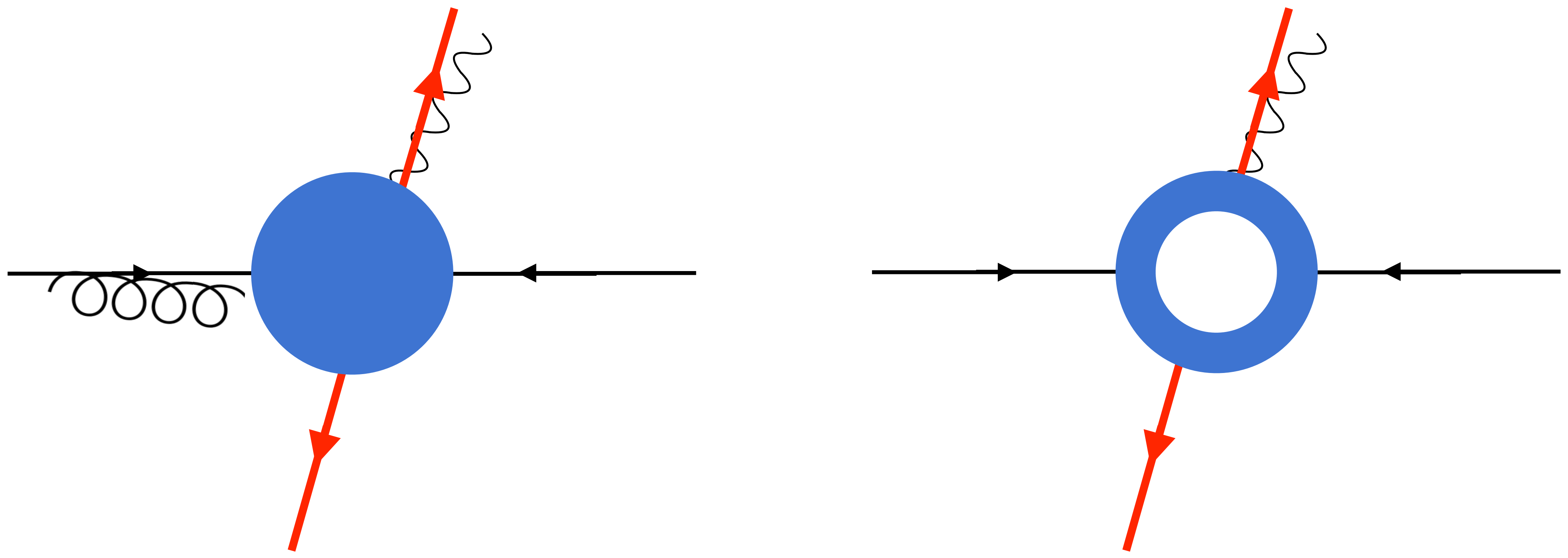


Final-state (collinear) radiation

There are configurations with $q_T > 0$ and **two unresolved emission if leptons are massless**



q_T-resummation QCD-QED(EW): massive final state

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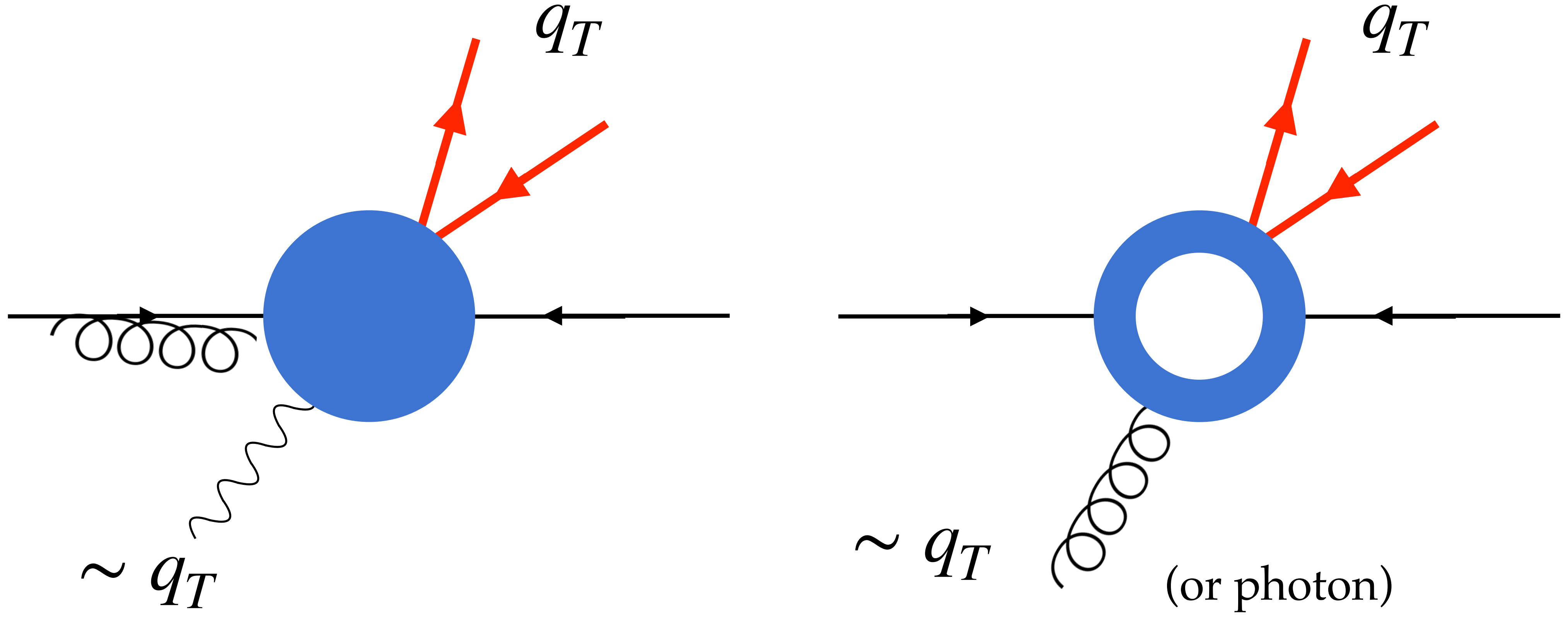

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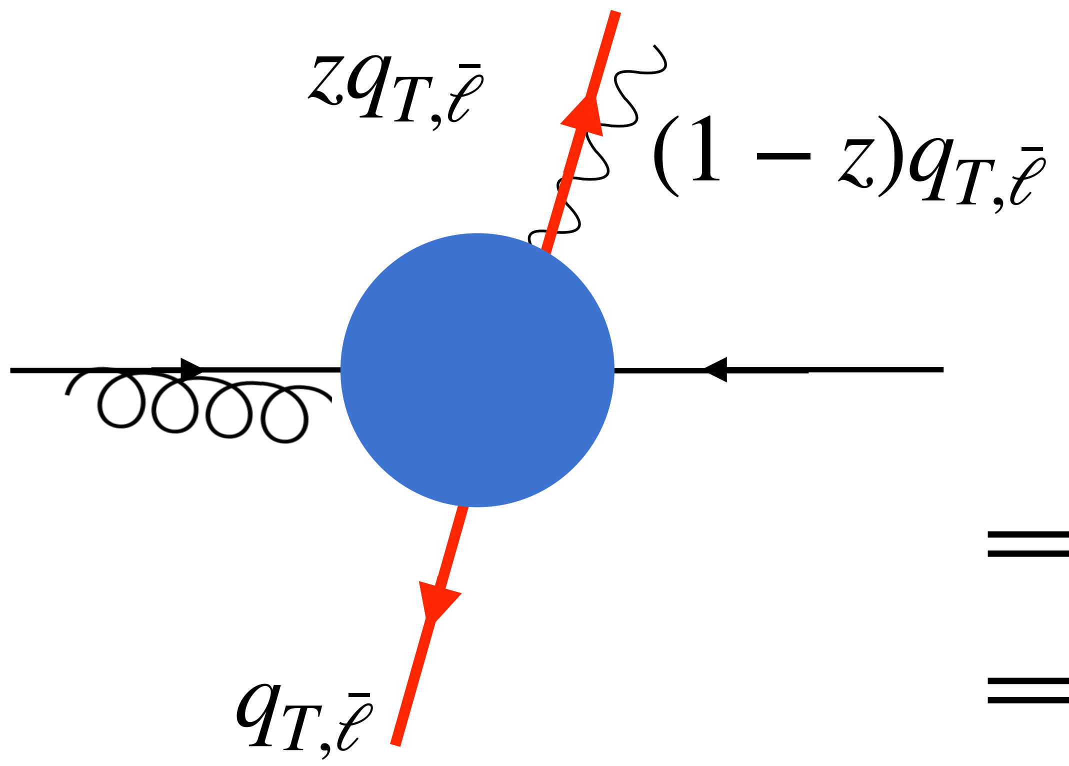

$$\mathbf{q}_{T,\bar{\ell}} + \mathbf{q}_{T,g} + \mathbf{q}_{T,\ell\gamma} = 0$$

$$\Longrightarrow -q_{T,\bar{\ell}} + zq_{T,\bar{\ell}} + (1-z)q_{T,\bar{\ell}} \sim 0$$

$$\Longrightarrow q_T = (1-z)q_{T,\bar{\ell}} > 0$$

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MAIN IDEA

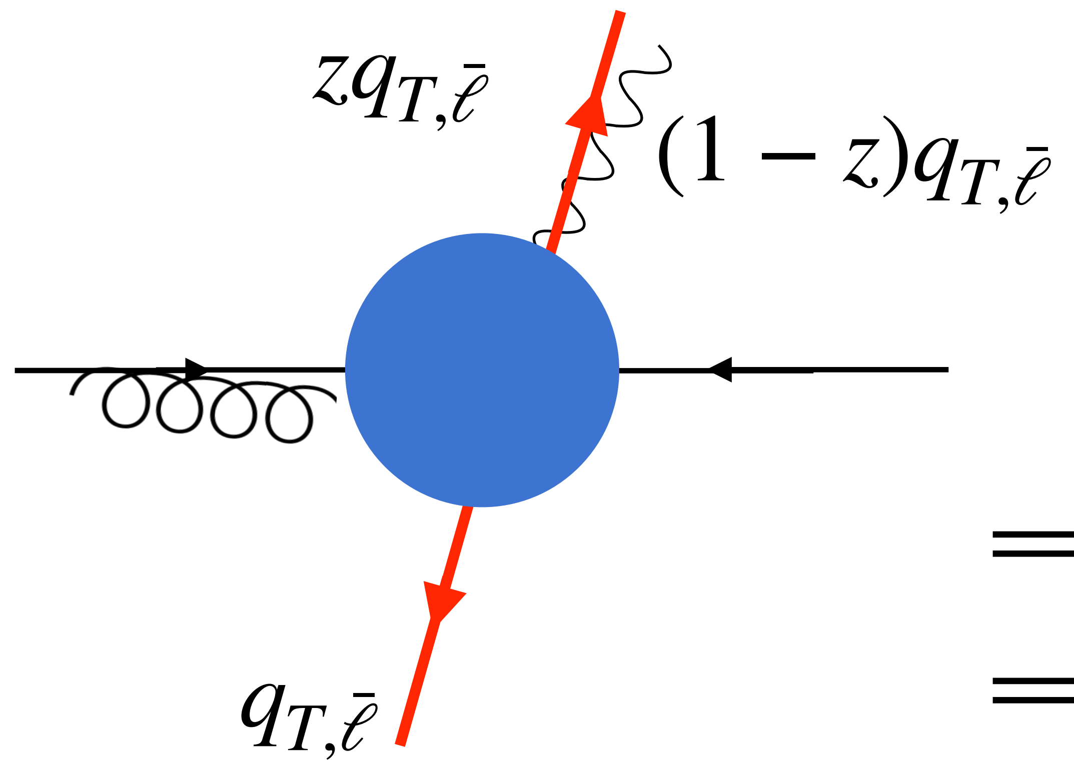
Consider **massive** leptons to resolve/regulate the singular limits associated to a photon collinear to a final-state lepton (or coloured massive particles)!

Same reasoning applies to **heavy-quark** production

→ Physics case: bare muons!

Final-state (collinear) radiation

There are configurations with $q_T > 0$ and **two unresolved emission if leptons are massless**



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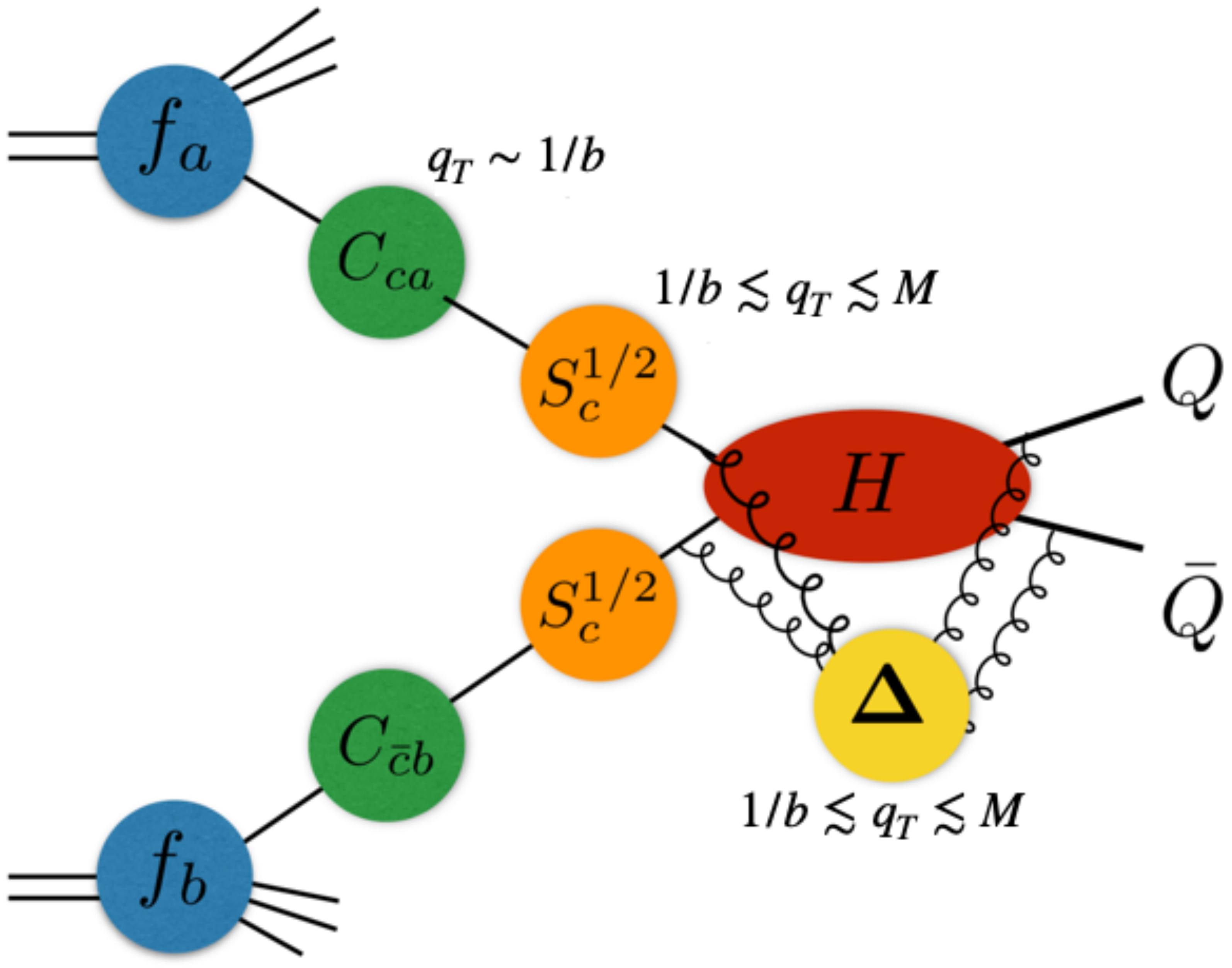
[Catani, Grazzini, Torre, 2014]

$$\frac{d\sigma^{(sing)}}{dQ^2 dY d\mathbf{q_T} d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},Q\bar{Q}}}{d\Omega}\int \frac{d\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}} S_c(Q,b) \sum_{a_1,a_2}\int_{x_1}^1 \frac{dz_1}{z_1}\int_{x_2}^1 \frac{dz_2}{z_2} [\mathrm{Tr}(\mathbf{H}^{Q\bar{Q}}\mathbf{\Delta})C_1 C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2)$$

See talks by J. Mazzitelli and S. Devoto



The resummation formula shows a **richer structure** because of additional soft singularities

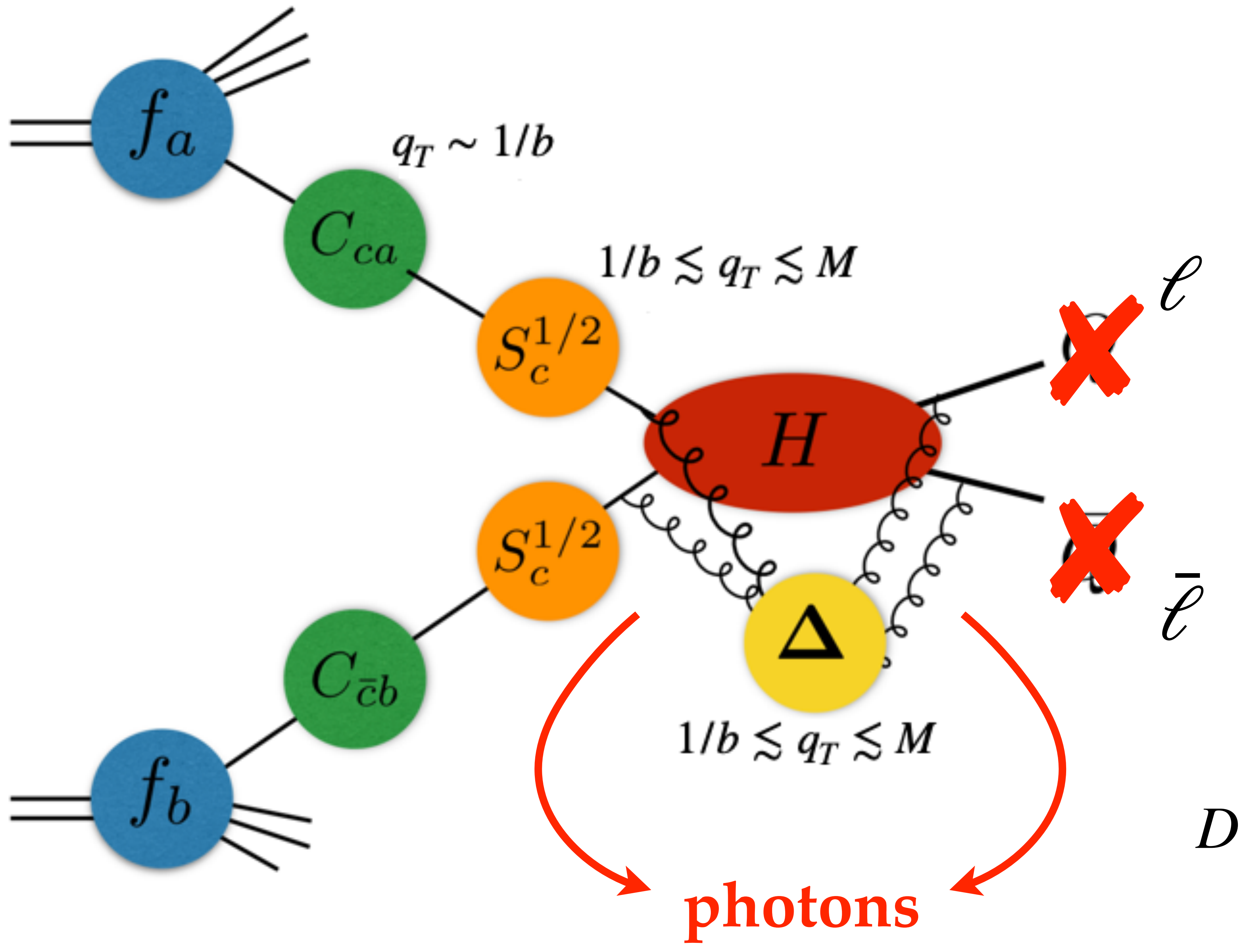
- Soft logarithms controlled by the **transverse momentum anomalous dimension Γ_t** known up to NNLO [Mitov, Sterman, Sung, 2009], [Neubert, et al 2009]
- Hard coefficient gets a **non-trivial** colour structure (matrix in colour-space). In general, resummation more involved because of colour evolution (path-ordered integral)
- Non-trivial azimuthal correlations [Catani, Grazzini, Sargsyan 2017]

q_T-resummation QCD-QED(EW): massive final state

[Catani, Grazzini, Torre, 2014]

$$\frac{d\sigma^{(sing)}}{dQ^2 dY d\mathbf{q_T} d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},Q\bar{Q}}}{d\Omega}\int \frac{d\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}} S_c(Q,b) \sum_{a_1,a_2}\int_{x_1}^1 \frac{dz_1}{z_1}\int_{x_2}^1 \frac{dz_2}{z_2} [\mathrm{Tr}(\mathbf{H}^{Q\bar{Q}}\mathbf{\Delta})C_1 C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2)$$

ABELIANISATION (only real emission of photons!)



Trivial colour structure

$$\mathrm{Tr}(\mathbf{H}^{Q\bar{Q}}\mathbf{\Delta}) \rightarrow \mathrm{H}^{\ell\bar{\ell}} \exp\left[-\int_{b_0^2/b^2}^{Q^2} dq^2 D'\left(\alpha(q^2), \Phi_B\right)\right]$$

A finite contribution is absorbed $H^{\ell\bar{\ell}}$

is like a B term, but it depends on kinematics

$$\beta = \sqrt{1 - 4\, m^2/s_{12}}$$

$$D^{'(1)}(\Phi_B) = -\, 2\left[e_{f(3)}e_{f(4)}\frac{1+\beta^2}{\beta}\ \ln\frac{1+\beta}{1-\beta} + \sum_{\ell=1}^{2}\sum_{k=3}^{4}\left(\frac{e^2_{f(k)}}{2} + e_{f(\ell)}e_{f(k)}\ln\frac{s^2_{\ell k}}{s_{12}\, m^2}\right)\right]$$

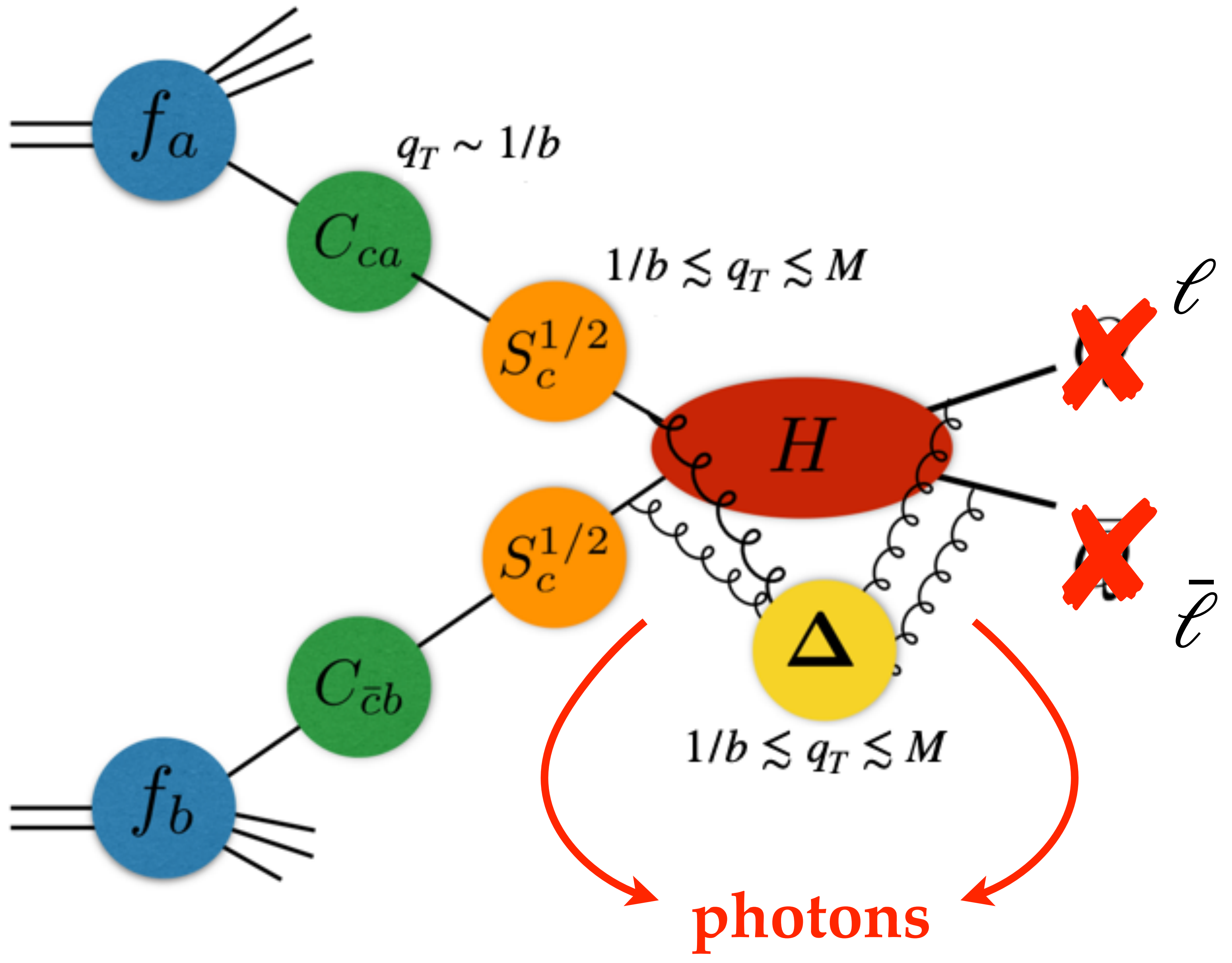
Explicit (large) logarithms of lepton mass in anomalous dimension

q_T-resummation QCD-QED(EW): massive final state

[Catani, Grazzini, Torre, 2014]

$$\frac{d\sigma^{(sing)}}{dQ^2 dY d\mathbf{q_T} d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},Q\bar{Q}}}{d\Omega}\int \frac{d\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}} S_c(Q,b) \sum_{a_1,a_2}\int_{x_1}^1 \frac{dz_1}{z_1}\int_{x_2}^1 \frac{dz_2}{z_2} [\mathrm{Tr}(\mathbf{H}^{Q\bar{Q}}\mathbf{\Delta})C_1 C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2)$$

ABELIANISATION (only real emission of photons!)



Trivial colour structure

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A finite contribution is absorbed $H^{\ell\bar{\ell}}$

is like a B term, but it depends on kinematics

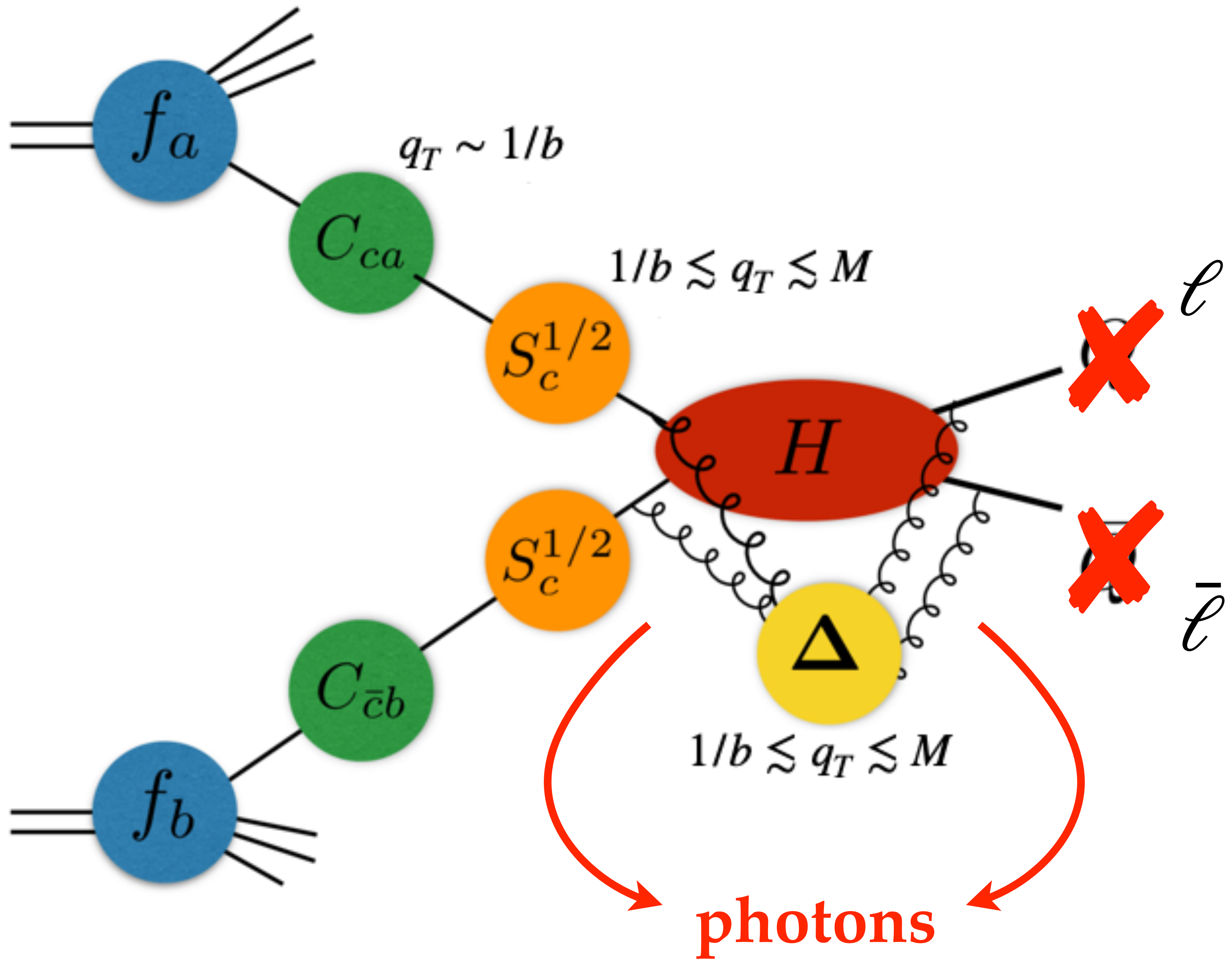
Dependence on flavour

$$c = \{q, \bar{q}, g\} \to c = \{u, d, \ldots, \bar{u}, \bar{d}, \ldots, g, \gamma\}$$

q_T-resummation QCD-QED(EW): massive final state

$$\frac{d\sigma^{(sing)}}{dQ^2 dY d\mathbf{q_T} d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},\ell\bar{\ell}}}{d\Omega}\int \frac{d\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}} S_c(Q,b,\Phi_b) \sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}[H^{\ell\bar{\ell}}C_1C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1,b_0^2/b^2) f_{a_2/h_2}(x_2,b_0^2/b^2)$$

ABELIANISATION (only real emission of photons!)



Expansion in two parameters

$$A = \sum_{k>0}\left(\frac{\alpha_S}{\pi}\right)^k A^{(k,0)} + \sum_{k>0}\left(\frac{\alpha}{\pi}\right)^k A^{(0,k)} + \sum_{j>0,k>0}\left(\frac{\alpha_S}{\pi}\right)^j\left(\frac{\alpha}{\pi}\right)^k A^{(j,k)}$$

Notice that they are **additive** at the exponent of the Sudakov form factor, so there is a multiplicative interplay when expanded at fixed order

The QED and mixed QCD-QED coefficients can be obtained by the corresponding pure QCD ones (Abelianisation)

q_T-resummation QCD-QED(EW): massive final state

$$\frac{d\sigma^{(sing)}}{dQ^2 dY d\mathbf{q_T} d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},\ell\bar{\ell}}}{d\Omega}\int \frac{d\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}} S_c(Q,b,\Phi_b) \sum_{a_1,a_2}\int_{x_1}^1 \frac{dz_1}{z_1}\int_{x_2}^1 \frac{dz_2}{z_2}[H^{\ell\bar{\ell}} C_1 C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2)$$

Treatment of the QED coupling

In fixed-order calculation, on-shell renormalisation of QED coupling constant and choice of best input parameters (no running coupling)

In resummation, ($\overline{MS}$) running coupling constant **coupled system of differential equations!**

[Cieri, Ferrera, Sborlini, 2018]
[Billis, Tackmann, Talbert, 2019]

$$\frac{d\ln\alpha_S(\mu^2)}{d\ln\mu} = \beta_{\rm QCD}(\alpha_S,\alpha) = -\sum_{j\geq 0}\left(\frac{\alpha_S}{\pi}\right)^{j+1}\beta^{(j,0)}_{\rm QCD} - \sum_{j\geq 0,k>0}\left(\frac{\alpha_S}{\pi}\right)^{j+1}\left(\frac{\alpha_S}{\pi}\right)^{k}\beta^{(j,k)}_{\rm QCD}$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu} = \beta_{\rm QED}(\alpha_S,\alpha) = -\sum_{j\geq 0}\left(\frac{\alpha}{\pi}\right)^{j+1}\beta^{(0,j)}_{\rm QED} - \sum_{j\geq 0,k>0}\left(\frac{\alpha_S}{\pi}\right)^{j+1}\left(\frac{\alpha_S}{\pi}\right)^{k}\beta^{(j,k)}_{\rm QED}$$

q_T-resummation QCD-QED(EW): massive final state

$$\frac{d\sigma^{(sing)}}{dQ^2 dY d\mathbf{q_T} d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},\ell\bar{\ell}}}{d\Omega}\int\frac{d\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}} S_c(Q,b,\Phi_b)\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}[H^{\ell\bar{\ell}}C_1C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1,b_0^2/b^2) f_{a_2/h_2}(x_2,b_0^2/b^2)$$

Treatment of the QED coupling $\sim \exp[Lg_1 + g_2 + \frac{\alpha_s}{\pi}g_3 + \cdots]$

[Cieri, Ferrera, Sborlini, 2018]

$$\mathcal{G}'_N(\alpha_S,\alpha,L) = \mathcal{G}_N(\alpha_S,L) + L\,g'^{(1)}(\alpha L) + g_N'^{(2)}(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g_N'^{(n)}(\alpha L)$$
$$+\, g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L,\alpha L)$$

We rederived this contribution find agreement with literature

Formally, this is the leading genuine mixed effect!
At fixed-order, first contribution $\mathcal{O}(\alpha_S^2\alpha)$ (or $\mathcal{O}(\alpha_S\alpha^2)$), and we found very small numerical impact at the LHC

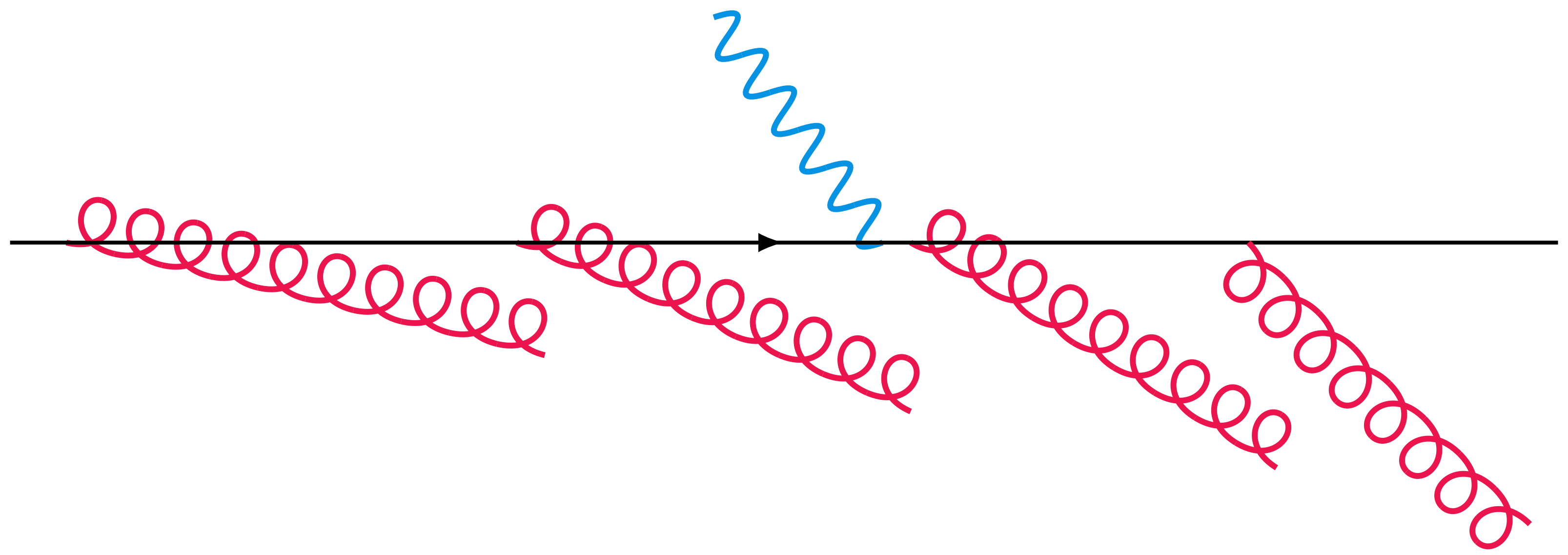
RadISH

[Bizon, Monni, Re, Rottoli, Torrielli '17]

Framework for numerical resummation

[Banfi, Salam, Zanderighi '05]
[Banfi, McAslan, Monni, Zanderighi '15]

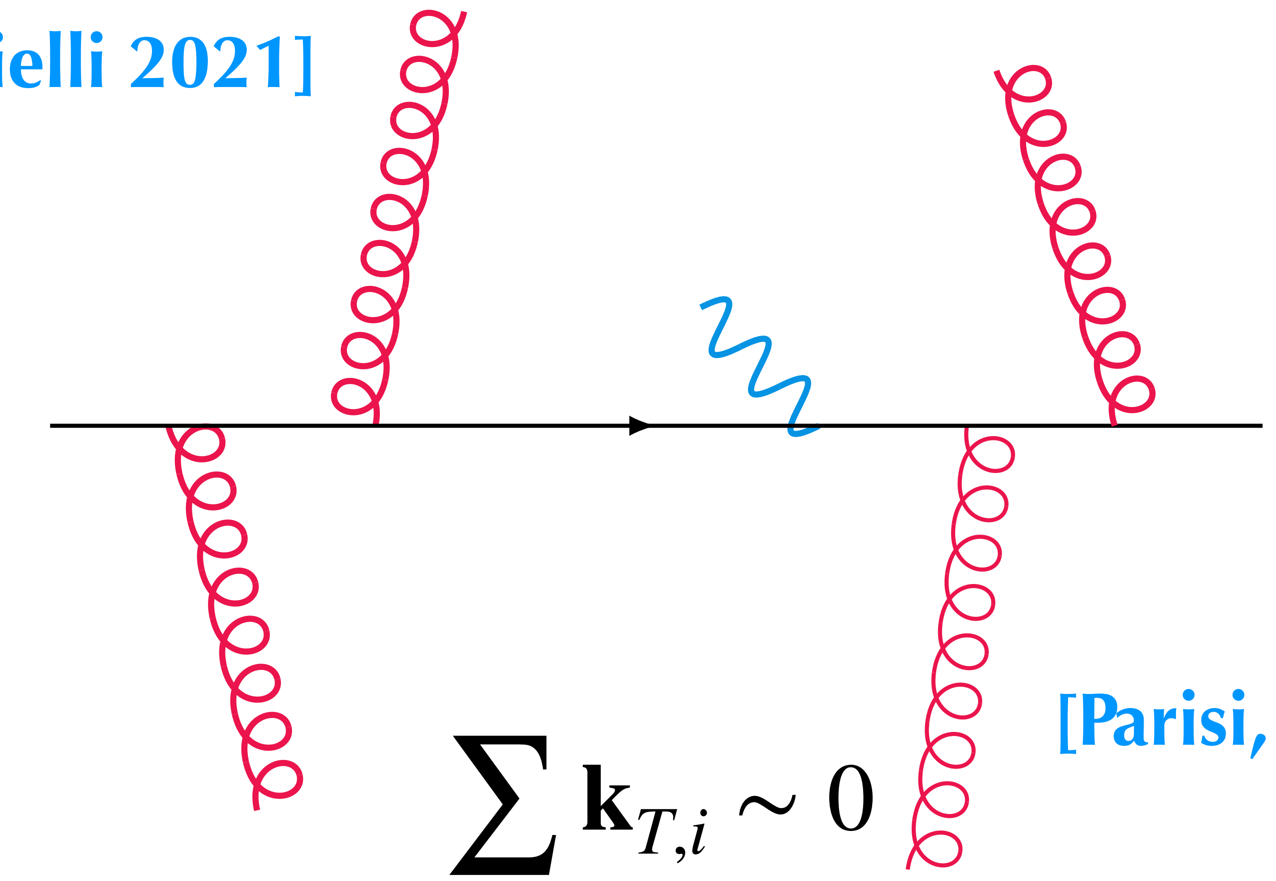
- for *global recursively infrared and collinear safe (rIRC) observables* (similar to CEASER / ARES)
- in **momentum (direct) space** for *vectorial* observables as the **transverse momentum**
- at very high logarithm accuracy (N^{3}LL$'$ accuracy) [Re, Rottoli, Torrielli 2021]



$$k_{T,i} \sim p_T \ll Q$$

Sudakov Limit
cross section suppresed as there is no phase space left for gluon emission

Exponential suppression



$$\sum_i \mathbf{k}_{T,i} \sim 0$$

[Parisi, Petronzio 1979]

Azimuthal cancellation
$p_T \ll Q$ away from the Sudakov region

Dominant at small p_T

Power suppression

RadISH in a nutshell (QCD)

Cumulant distribution for an observable V in color singlet production (hard scale M, resummation scale Q)

LL NLL NNLL

$$\ln \Sigma(v) = \ln \int d\sigma\, \Theta(v - V) \sim \sum_n \left[\mathcal{O}\left(\alpha_s^n \ln^{n-1}(1/v)\right) + \mathcal{O}\left(\alpha_s^n \ln^{n}(1/v)\right) + \mathcal{O}\left(\alpha_s^n \ln^{n-1}(1/v)\right) + \ldots \right]$$

See talks by A. Soto-Ontoso

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{T,1}}{k_{T,1}} \mathcal{L}(k_{T,1})\, e^{-R(k_{T,1})}\, \mathcal{F}(v, \Phi_B, k_{T,1})$$

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$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{T,1}}{k_{T,1}} \mathcal{L}(k_{T,1})\, e^{-R(k_{T,1})}\, \mathcal{F}(v, \Phi_B, k_{T,1})$$

RADIATOR (for the ordering variable)

$$R(k_{T,1}) = \sum_{\ell=1}^{2} R_\ell(k_{T,1}) = -L g_1(\lambda) + \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n g_{n+2}(\lambda)$$

$$L = \ln \frac{Q}{k_{T,1}}, \quad \lambda = \alpha_s \beta_0 L$$

$$R_\ell(k_{T,1}) = \int_{k_{T,1}}^{M} \frac{dq}{q} \left[A_\ell\left(\alpha_s(q)\right) \ln \frac{M^2}{q^2} + B_\ell\left(\alpha_s(q)\right) \right]$$

flavour-conserving soft-collinear and hard-collinear **anomalous dimensions**

Connection with b-space resummation

RadISH in a nutshell (QCD)

Cumulant distribution for an observable V in color singlet production (hard scale M, resummation scale Q)

LL NLL NNLL

$$\ln \Sigma(v) = \ln \int d\sigma \, \Theta(v - V) \sim \sum_n \left[\mathcal{O}\left(\alpha_s^n \ln^{n-1}(1/v)\right) + \mathcal{O}\left(\alpha_s^n \ln^{n}(1/v)\right) + \mathcal{O}\left(\alpha_s^n \ln^{n-1}(1/v)\right) + \ldots \right]$$

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{T,1}}{k_{T,1}} \mathscr{L}(k_{T,1}) \, e^{-R(k_{T,1})} \, \mathscr{F}(v, \Phi_B, k_{T,1})$$

Parton densities

LUMINOSITY

$$\mathscr{L}(k_{T,1}) = \sum_{c,d} |M_B|^2_{cd} \sum_i \left[C_{c\,i} \otimes f_i(k_{T,1}) \right](x_1) \sum_j \left[C_{d\,j} \otimes f_j(k_{T,1}) \right](x_2) \; H(\mu_R)$$

Born matrix element Collinear functions Hard-virtual corrections

RadISH in a nutshell (QCD)

Cumulant distribution for an observable V in color singlet production (hard scale M, resummation scale Q)

$$\ln \Sigma(v) = \ln \int d\sigma \, \Theta(v - V) \sim \sum_n \Big[\underbrace{\mathcal{O}\left(\alpha_s^n \ln^{n-1}(1/v)\right)}_{\text{LL}} + \underbrace{\mathcal{O}\left(\alpha_s^n \ln^{n}(1/v)\right)}_{\text{NLL}} + \underbrace{\mathcal{O}\left(\alpha_s^n \ln^{n-1}(1/v)\right)}_{\text{NNLL}} + \ldots \Big]$$

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{T,1}}{k_{T,1}} \mathcal{L}(k_{T,1}) \, e^{-R(k_{T,1})} \mathcal{F}(v, \Phi_B, k_{T,1})$$

RESOLVED REAL RADIATION

- ensamble of soft and collinear partons
- **encodes** the full dependence on the observable V
- **finite** in four dimensions and implemented as a **shower** of primary emissions off the initial-state legs, ordered in transverse momentum.

Illuminating RadISH with QED (EW)

GOAL: combining higher-order QCD resummation with the resummation of EW and mixed QCD-EW effects

$$\mathcal{O}\left(\alpha^n \ln^{n+1}(1/v)\right) + \mathcal{O}\left(\alpha^n \ln^{n}(1/v)\right)$$ NLL in EW (+prime)

$$\mathcal{O}\left(\alpha_s^n \alpha^m \ln^{n+m}(1/v)\right)$$ NLL in QCD-EW (+prime)

Physical picture: the same ensemble of soft and collinear partons (shower) can be of pure QCD, pure QED or mixed QCD-QED nature according to the assigned Monte Carlo weight.
Exploit the connection with b-space resummation

RADIATOR

$$e^{-R(k_{t,1})} \to e^{-\left[R(k_{t,1}) + R^{\text{QED}}(k_{t,1}) + R^{\text{MIX}}(k_{t,1})\right]}$$

$$R^{\text{QED}}(k_{T,1}) = \int_{k_{T,1}}^{M} \frac{dq}{q} \left\{ \sum_{\ell=1}^{2} \left[A'_\ell \left(\alpha_s(q)\right) \ln \frac{M^2}{q^2} + B'_\ell \left(\alpha_s(q)\right) \right] + D'\left(\alpha(q), \Phi_B\right) \right\}$$

Anomalous dimensions associated to the two incoming **massless** legs

Initial-Final and Final-Final **soft wide angle correlations**, logarithmically enhanced in the **lepton mass**

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Exploit the connection with b-space resummation

RADIATOR $$e^{-R(k_{t,1})} \to e^{-\left[R(k_{t,1}) + R^{\mathrm{QED}}(k_{t,1}) + R^{\mathrm{MIX}}(k_{t,1})\right]}$$

Coupled evolutions of running couplings $$\xi = 1 - 2\,\alpha_s\,\beta_0 \ln\frac{\mu_{\mathrm{R}}}{q}, \quad \xi' = 1 - 2\,\alpha\,\beta_0' \ln\frac{\mu_R}{q}$$

$$R^{\mathrm{MIX}}(k_{t1}) = -\frac{1}{2\pi}\sum_{\ell=1}^{2}\int_{k_{t1}}^{M}\frac{dq}{q}\left[\frac{\alpha_s^2\beta_{01}\ln\xi'}{\xi^2\beta_0'}A_\ell^{(1)} + \frac{\alpha^2\beta_{01}'\ln\xi}{\xi'^2\beta_0}A_\ell^{'(1)}\right]\ln\frac{M^2}{q^2} = -\,g_{11}(\lambda,\lambda') - \,g_{11}'(\lambda,\lambda')$$

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Exploit the connection with b-space resummation

RADIATOR

$$e^{-R(k_{t,1})} \rightarrow e^{-\left[R(k_{t,1}) + R^{\mathrm{QED}}(k_{t,1}) + R^{\mathrm{MIX}}(k_{t,1})\right]}$$

We **keep a subleading term** to have all the ingredients required for matching at $\mathcal{O}(\alpha_s \alpha)$

$$e^{-R(k_{t,1})} \rightarrow e^{-\left[R(k_{t,1}) + R^{\mathrm{QED}}(k_{t,1}) + R^{\mathrm{MIX}}(k_{t,1}) + \frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi} B^{(1,1)} L\right]}$$

$$\mathcal{O}\left(\alpha_s^n \alpha^m \ln^{n+m-1}(1/v)\right)$$

Illuminating RadISH with QED (EW)

GOAL: combining higher-order QCD resummation with the resummation of EW and mixed QCD-EW effects at

$$\mathcal{O}\left(\alpha^n \ln^{n+1}(1/v)\right) + \mathcal{O}\left(\alpha^n \ln^n(1/v)\right)$$ NLL in EW (+prime)

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Physical picture: the same ensemble of soft and collinear partons (shower) can be of pure QCD, pure QED or mixed QCD-QED nature according to the assigned Monte Carlo weight.
Exploit the connection with b-space resummation

LUMINOSITY

$$\mathcal{L}(k_{T,1}) = \sum_{c,d} |M_B|^2_{cd} \sum_i \left[C_{c\,i} \otimes f_i(k_{T,1})\right](x_1) \sum_j \left[C_{d\,j} \otimes f_j(k_{T,1})\right](x_2)\; H(\mu_R)$$

PDF sets including the **photons** and **QED effects in evolution**

Illuminating RadISH with QED (EW)

GOAL: combining higher-order QCD resummation with the resummation of EW and mixed QCD-EW effects at

$$\mathcal{O}\left(\alpha^n \ln^{n+1}(1/v)\right) + \mathcal{O}\left(\alpha^n \ln^{n}(1/v)\right)$$ NLL in EW (+prime)

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Exploit the connection with b-space resummation

LUMINOSITY

$$\mathcal{L}(k_{T,1}) = \sum_{c,d} |M_B|^2_{cd} \sum_i \left[C_{c\,i} \otimes f_i(k_{T,1})\right](x_1) \sum_j \left[C_{d\,j} \otimes f_j(k_{T,1})\right](x_2)\, H(\mu_R)$$

$$C_{ab}(z) \to C_{ab}(z) + \frac{\alpha}{2\pi} C_{ab}^{\prime(1)}(z) + \frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi} C_{ab}^{(1,1)}(z)$$ $$\mathcal{O}\left(\alpha_s^n \ln^{n-1}(1/v)\right)$$

$$H(\mu_R) \to H(\mu_R) + \frac{\alpha}{2\pi} H_{cd}^{\prime(1)}(\mu_R) + \frac{\alpha}{2\pi} F^{\prime(1)}(\Phi_B) + \frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi} H_{cd}^{(1,1)}(\mu_R)$$ $$\mathcal{O}\left(\alpha_s^n \alpha^m \ln^{n+m-2}(1/v)\right)$$

Illuminating RadISH with QED (EW)

Extension of **RESUMMATION** and **EXPANSION** modules in RadISH

- QED introduces a dependence on the flavor of the Born subprocesses: separate shower for each channel
- For Neutral Current DY, additional $\gamma\gamma$ process included (up to $\mathrm{NLL}'_{\mathrm{EW}}$ accuracy)
- $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha_s\alpha)$ splitting kernels and collinear coefficient functions implemented via HOPPET (qed branch)
- Two-loop QCD-EW amplitudes computed in pole appoximation
- One-loop EW amplitudes evaluated with RECOLA2, all tree-level matrix elements computed analytically

(Additive) **Matching** to fixed order obtained with MATRIX

$$\frac{d\sigma}{dp_T^{\ell\ell}} = \frac{d\sigma_{\mathrm{RES}}}{dp_T^{\ell\ell}} + \frac{d\sigma_{\mathrm{FO}}}{dp_T^{\ell\ell}} - \left[\frac{d\sigma_{\mathrm{RES}}}{dp_T^{\ell\ell}}\right]_{\mathrm{FO}}$$

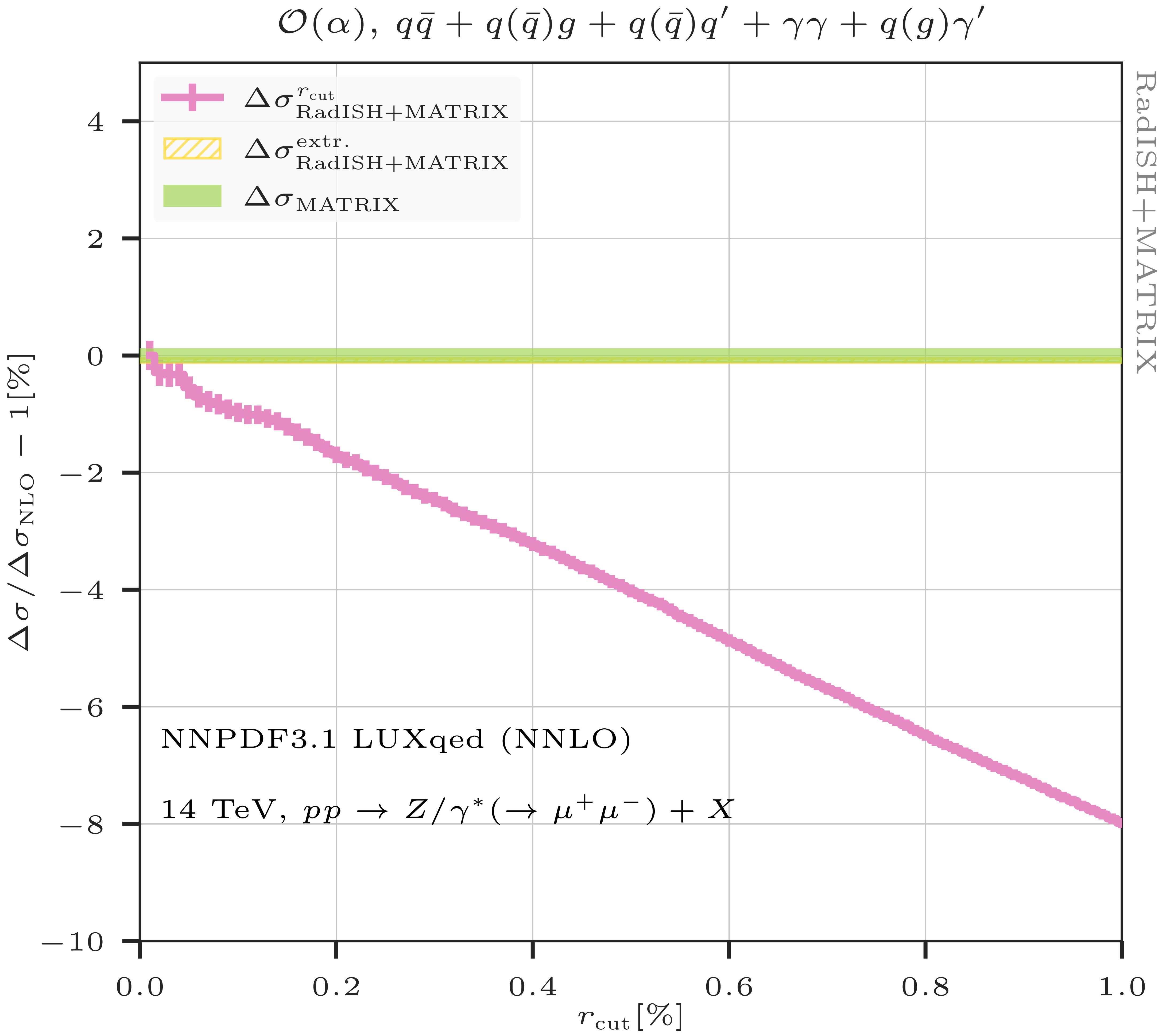
Our best prediction $\mathrm{N^3LL}'_{\mathrm{QCD}} + \mathrm{N^2LO}_{\mathrm{QCD}} + \mathrm{NLL}'_{\mathrm{EW}} + \mathrm{NLO}_{\mathrm{EW}} + \mathrm{nNLL}'_{\mathrm{MIX}}$

Validation against fixed-order results

The matching provides another independent implementation of q_T-slicing: non-trivial validation of the new perturbative ingredients

SETUP - NC DY (LHC @ $\sqrt{s} = 14$ TeV)

- `NNPDF31_nnlo_as_0118_luxqed`
- $p_{T,\mu} > 25$ GeV, $|y_\mu| < 2.5$, $m_{\mu\mu} > 50$ GeV
- massive muons (no photon lepton recombination)
- G_μ scheme, complex mass scheme
- fixed scale $\mu_F = \mu_R = m_{\mu\mu}$



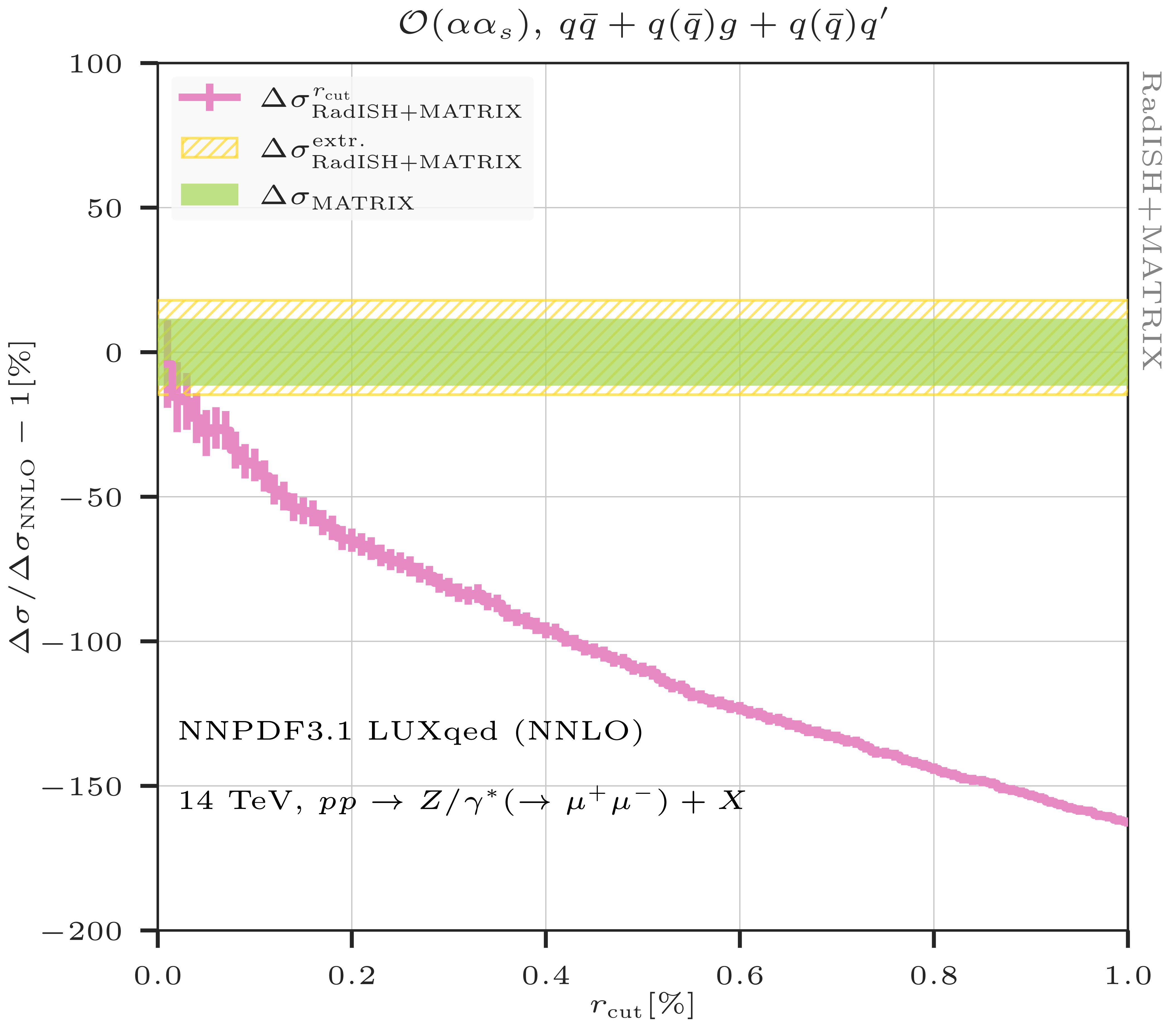
Perfect agreement for the NLO EW corrections at the level of the fiducial cross section
(reference prediction obtained with dipole subtraction as implemented in MATRIX)

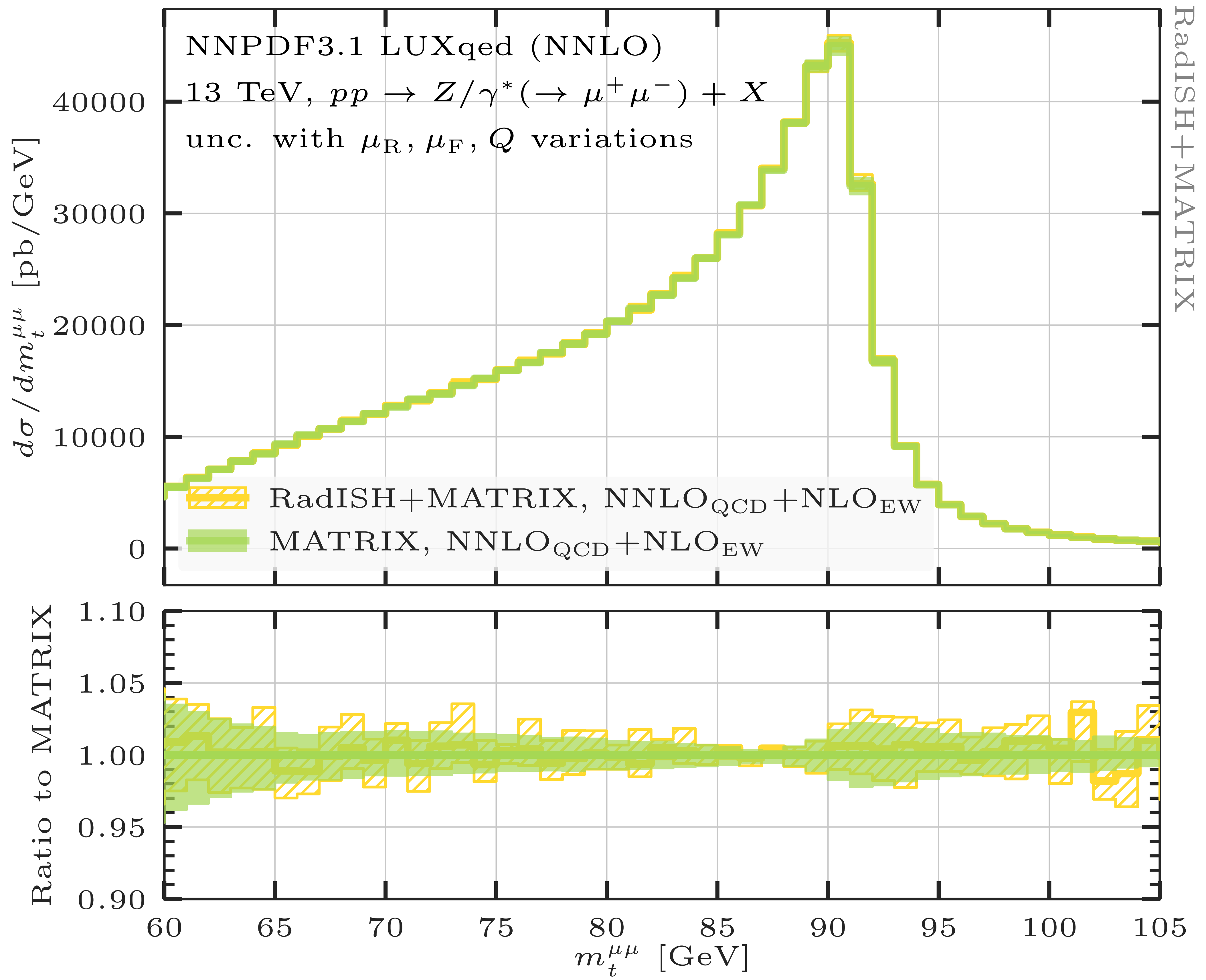
Linear power corrections in the slicing parameter $r_{\rm cut} = q_T^{\mu\mu}/m_{\mu\mu}$ mainly due to soft radiation off the massive final states

Validation against fixed-order results [Bonciani, LB, Grazzini, Kallweitt, Rana, Tramontano, Vicini '21]

Validation at higher orders

- $\mathcal{O}(\alpha_s \alpha)$ correction for fiducial cross sections (sum over all channels)
- Going differentially: invariant mass distributions at $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$





First "pheno" studies

RadISH+Matrix predictions

- $\mathrm{N^3LL'_{QCD}} + \mathrm{N^2LO_{QCD}}$ (**pure QCD model,** `NNPDF31_nnlo_as_0118`)
- $\mathrm{NLL'_{QCD}} + \mathrm{NLO_{QCD}} + \mathrm{NLL'_{EW}} + \mathrm{NLO_{EW}}$
- $\mathrm{N^3LL'_{QCD}} + \mathrm{N^2LO_{QCD}} + \mathrm{NLL'_{EW}} + \mathrm{NLO_{EW}} + \mathrm{nNLL'_{MIX}}$
- Caveat: not matched at $\mathcal{O}(\alpha_s \alpha)$

$$\frac{d\sigma}{dp_T^{\ell\ell}} = \frac{d\sigma_{\rm FO}}{dp_T^{\ell\ell}} + Z(p_T^{\ell\ell})\left\{\frac{d\sigma_{\rm RES}}{dp_T^{\ell\ell}} + - \left[\frac{d\sigma_{\rm RES}}{dp_T^{\ell\ell}}\right]_{\rm FO}\right\}$$

$$Z(p_t^{\ell\ell}) = \left[1 - \left(p_t^{\ell\ell}/p_{t0}\right)^u\right]^h \Theta(p_{t0} - p_t^{\ell\ell})$$

Uncertainty bands obtained as envelope of

- 7-point variation of μ_R, μ_F, at fixed $Q = m_{\ell\ell}/2$
- Plus 2-point variation of Q, at fixed $\mu_R = \mu_F = m_{\ell\ell}$
- Times 3-point variation of p_{t0} in $\{2/3, 1, 3/2\} \times m_V$

First "pheno" studies

RadISH+Matrix predictions

- $\mathrm{N^3LL'_{QCD}} + \mathrm{N^2LO_{QCD}}$ (**pure QCD model,** `NNPDF31_nnlo_as_0118`)
- $\mathrm{NLL'_{QCD}} + \mathrm{NLO_{QCD}} + \mathrm{NLL'_{EW}} + \mathrm{NLO_{EW}}$
- $\mathrm{N^3LL'_{QCD}} + \mathrm{N^2LO_{QCD}} + \mathrm{NLL'_{EW}} + \mathrm{NLO_{EW}} + \mathrm{nNLL'_{MIX}}$
- Caveat: not matched at $\mathcal{O}(\alpha_s\alpha)$

Compared with
(no hadronization, no MPI, AZNLO tune)

- $\mathrm{PWG_{EW}}$+PY8+PHOTOS: include NLO QCD + NLO EW with massive leptons (and factorized mixed contributions) [Barze, Chiesa, Montagna, Nason, Nicrosini, Piccinini, Vicini]
- $\mathrm{PWG_{QCD}}$+PY8+PHOTOS: Simple NLO QCD + PS generator interfaced with PHOTOS to include FSR QED [Alioli, Nason, Oleari, Re '08]

First “pheno” studies

RadISH+Matrix predictions

- $\text{N}^3\text{LL}'_{\text{QCD}} + \text{N}^2\text{LO}_{\text{QCD}}$ **(pure QCD model,** `NNPDF31_nnlo_as_0118`)
- $\text{NLL}'_{\text{QCD}} + \text{NLO}_{\text{QCD}} + \text{NLL}'_{\text{EW}} + \text{NLO}_{\text{EW}}$
- $\text{N}^3\text{LL}'_{\text{QCD}} + \text{N}^2\text{LO}_{\text{QCD}} + \text{NLL}'_{\text{EW}} + \text{NLO}_{\text{EW}} + \text{nNLL}'_{\text{MIX}}$
- Caveat: not matched at $\mathcal{O}(\alpha_s \alpha)$

SETUP - NC DY (LHC @ $\sqrt{s} = 13$ TeV)

- `NNPDF31_nnlo_as_0118_luxqed`
- $p_{T,\mu} > 27$ GeV, $|y_\mu| < 2.5$, 66 GeV $< m_{\mu\mu} <$ 116 GeV
- **massive muons** (no photon lepton recombination)
- G_μ scheme, complex mass scheme
- fixed scale $\mu_F = \mu_R = m_{\mu\mu}$

Compared with
(no hadronization, no MPI, AZNLO tune)

- PWG$_{\text{EW}}$+PY8+PHOTOS: include NLO QCD + NLO EW with massive leptons (and factorized mixed contributions)
- PWG$_{\text{QCD}}$+PY8+PHOTOS: Simple NLO QCD + PS generator interfaced with PHOTOS to include FSR QED

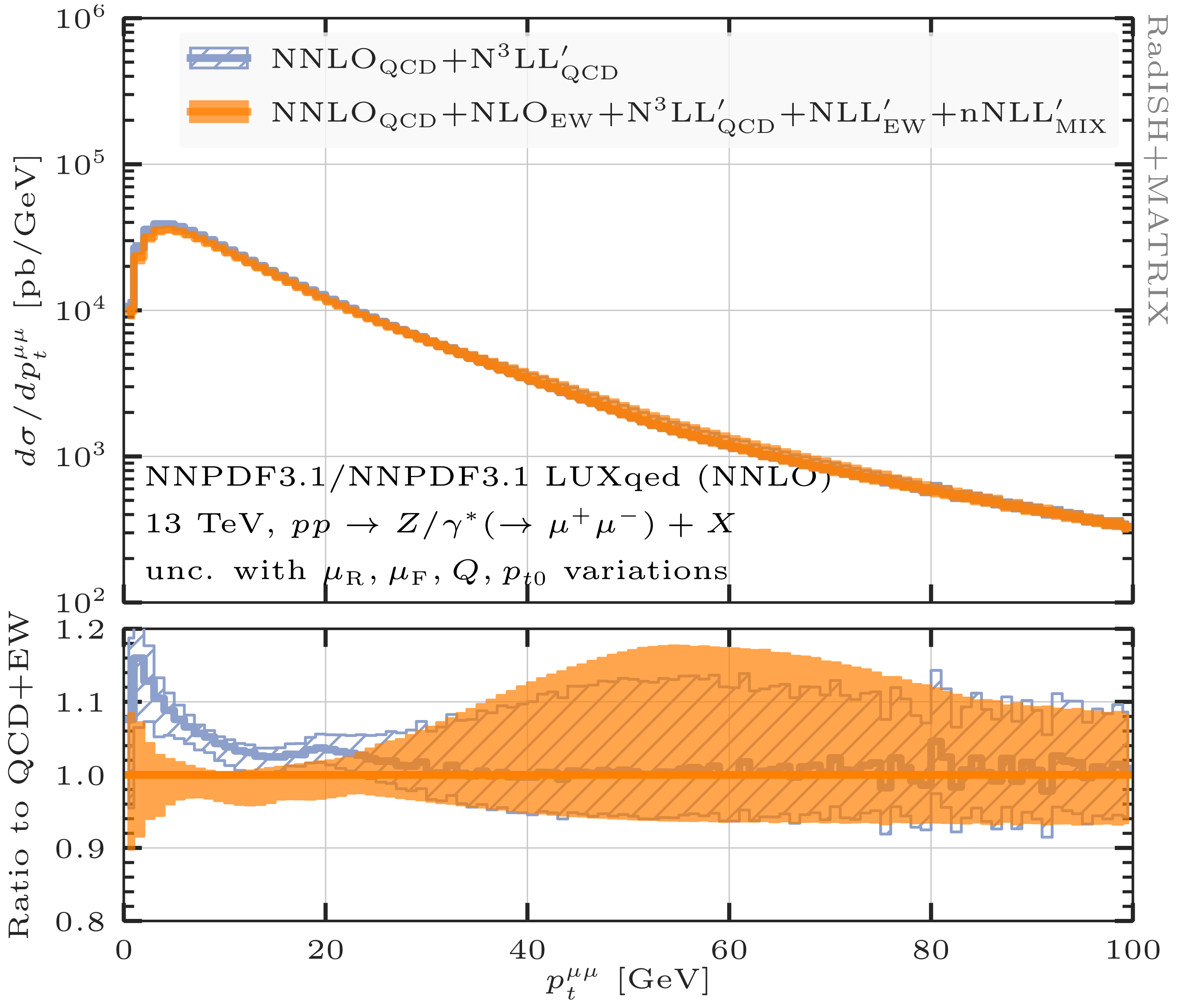
SETUP - CC DY (LHC @ $\sqrt{s} = 13$ TeV)

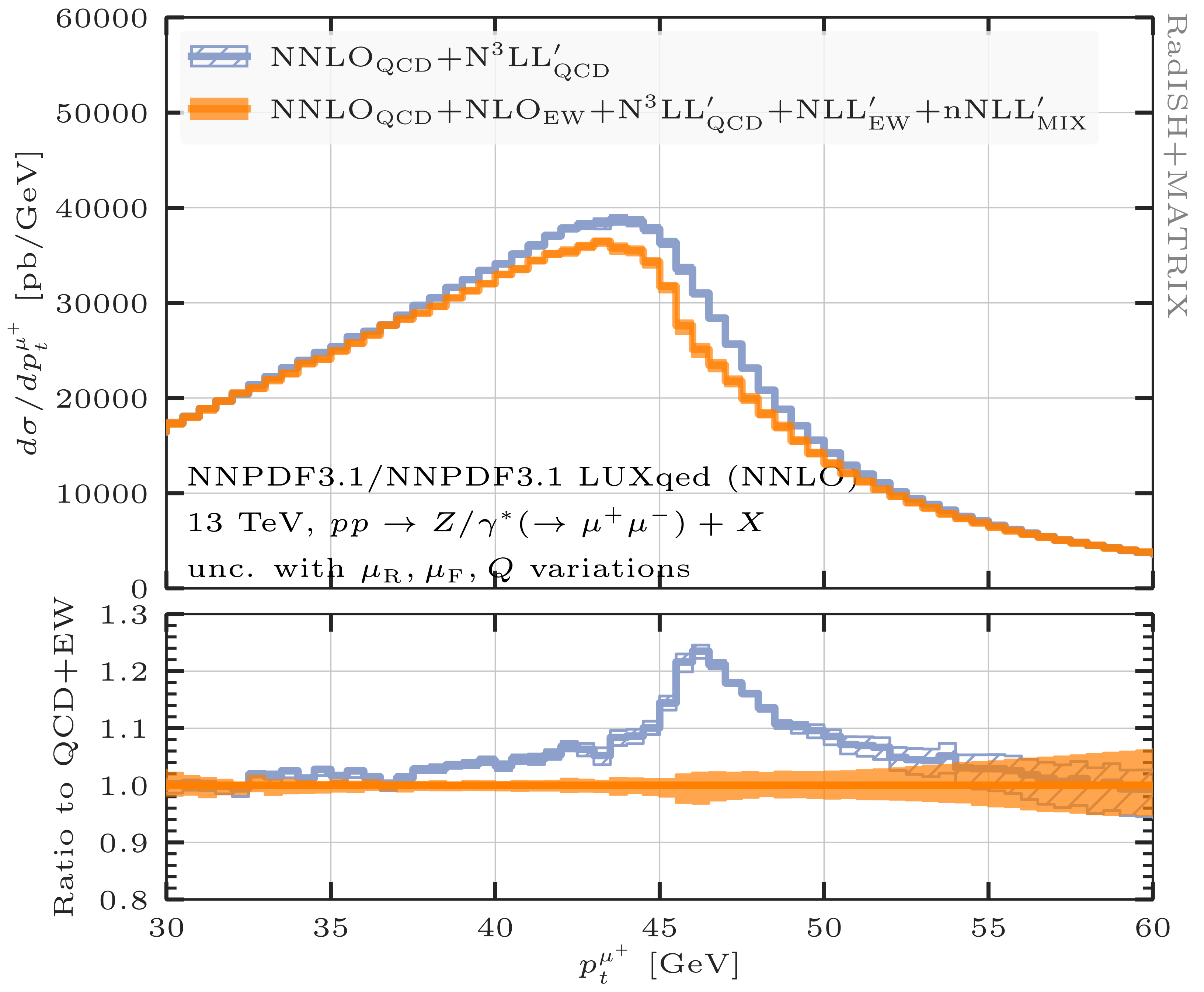
- `NNPDF31_nnlo_as_0118_luxqed`
- 26 GeV $< p_{T,\mu} <$ 55 GeV, $|y_\mu| < 2.4$, $m_T^{\mu\nu} = \sqrt{2 p_T^\mu p_T^{\nu (1 - cos\Delta\Phi^{\mu\nu})}} > 40$ GeV
- massive muons (no photon lepton recombination)
- G_μ scheme, complex mass scheme
- fixed scale $\mu_F = \mu_R = \sqrt{m_{\mu\nu}^2 + (p_T^{\mu\nu})^2}$

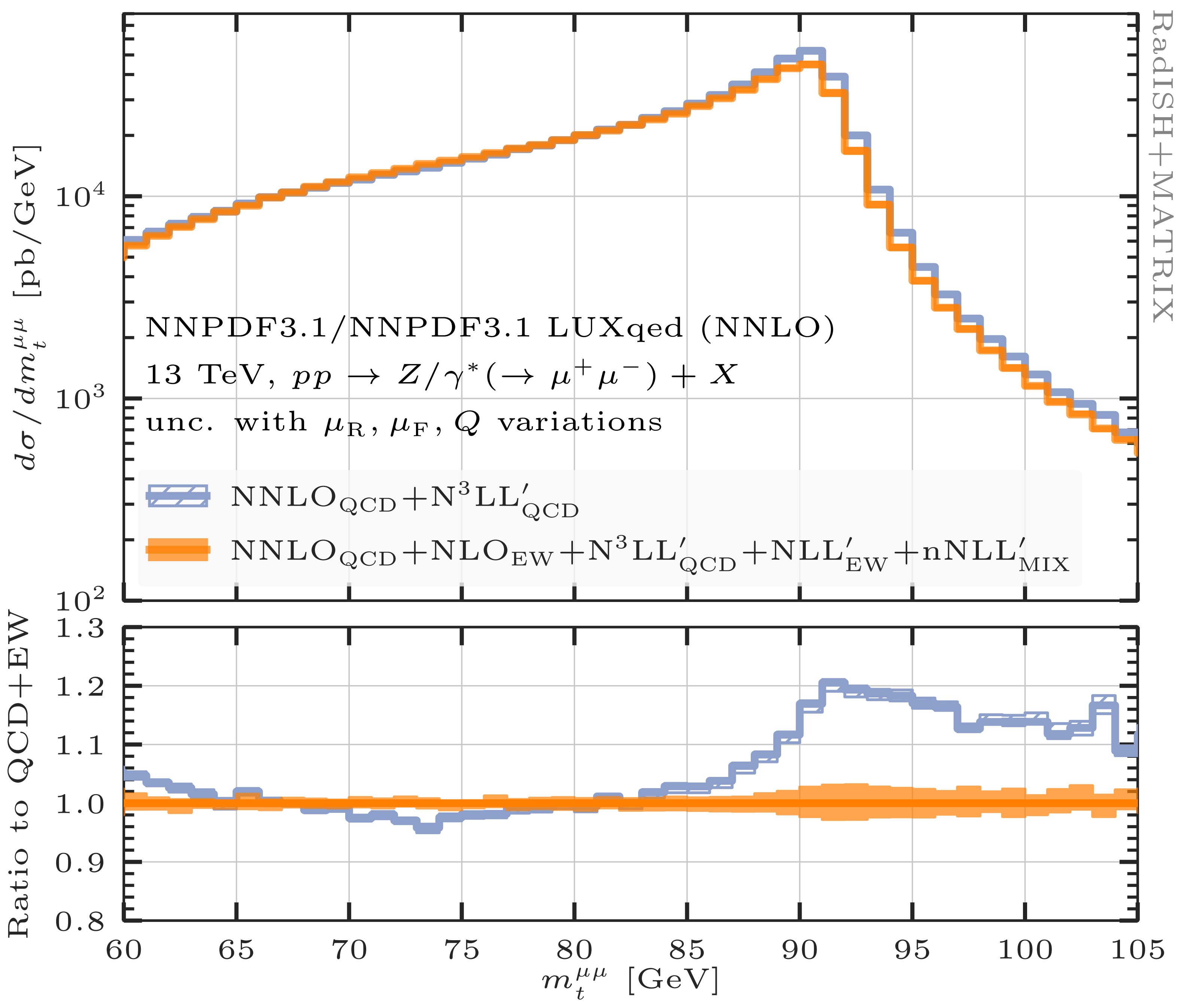
Phenomenology impact on NC DY

Impact of EW

- Large effects in observables where either all-order resummation or radiative kinematic effects are relevant
- FSR QED drives the impact of the corrections
- EW effects exceed the uncertainty bands of the pure QCD model: not unexpected as they are genuine “new” contributions



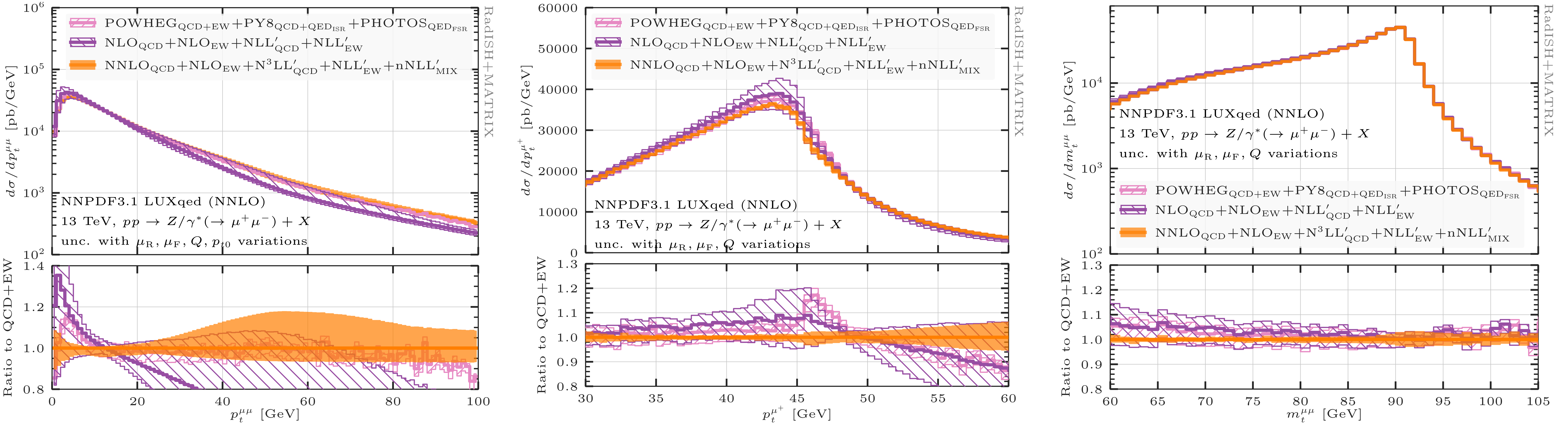




Phenomenology impact on NC DY

Comparison with PWG$_{\text{EW}}$+PY8+PHOTOS / $\text{NLL}'_{\text{QCD}} + \text{NLO}_{\text{QCD}} + \text{NLL}'_{\text{EW}} + \text{NLO}_{\text{EW}}$

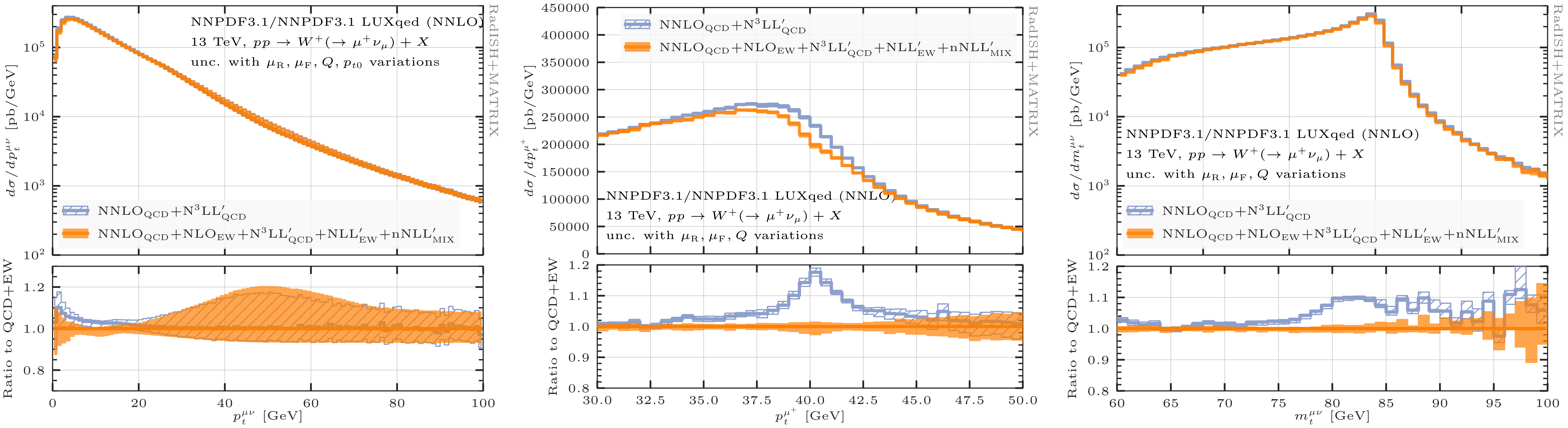
- PWG$_{\text{EW}}$+PY8+PHOTOS: uncertainty bands only scale variations (no estimate of resummation uncertainties)
- Good agreement for Born observables with some shape distortion at the Jacobian peak of $p_T^{\mu^+}$
- Relative good agreement at small transverse momentum, large differences in the transition region; delayed matching to fixed order result for PWG$_{\text{EW}}$+PY8+PHOTOS (accidentally closer to the higher-order result)



Phenomenology impact on CC DY

Impact of EW

- Similar results for the CC DY
- The impact of EW is smaller: this can be related to the fact that now we have only one charged lepton in the final state

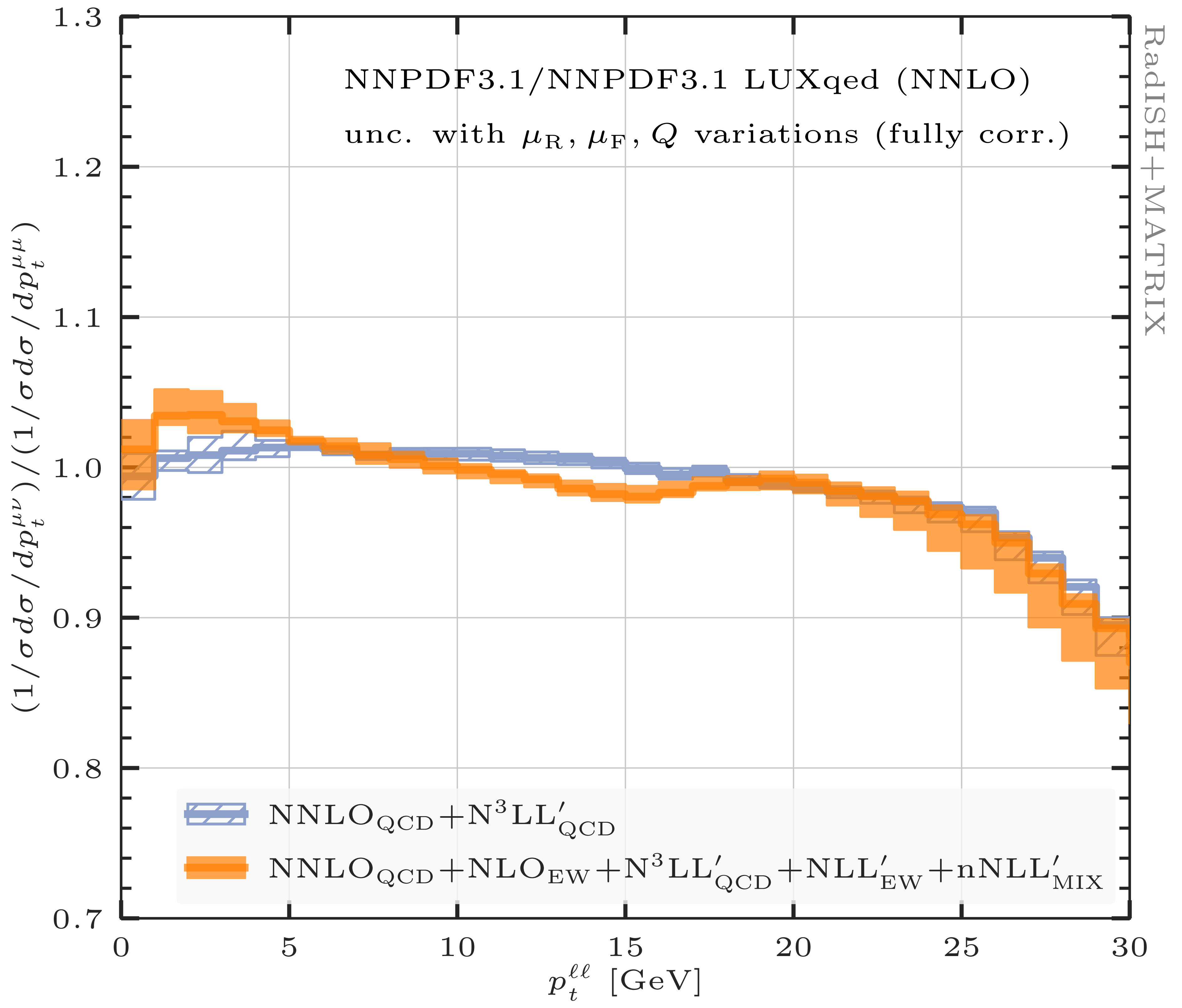


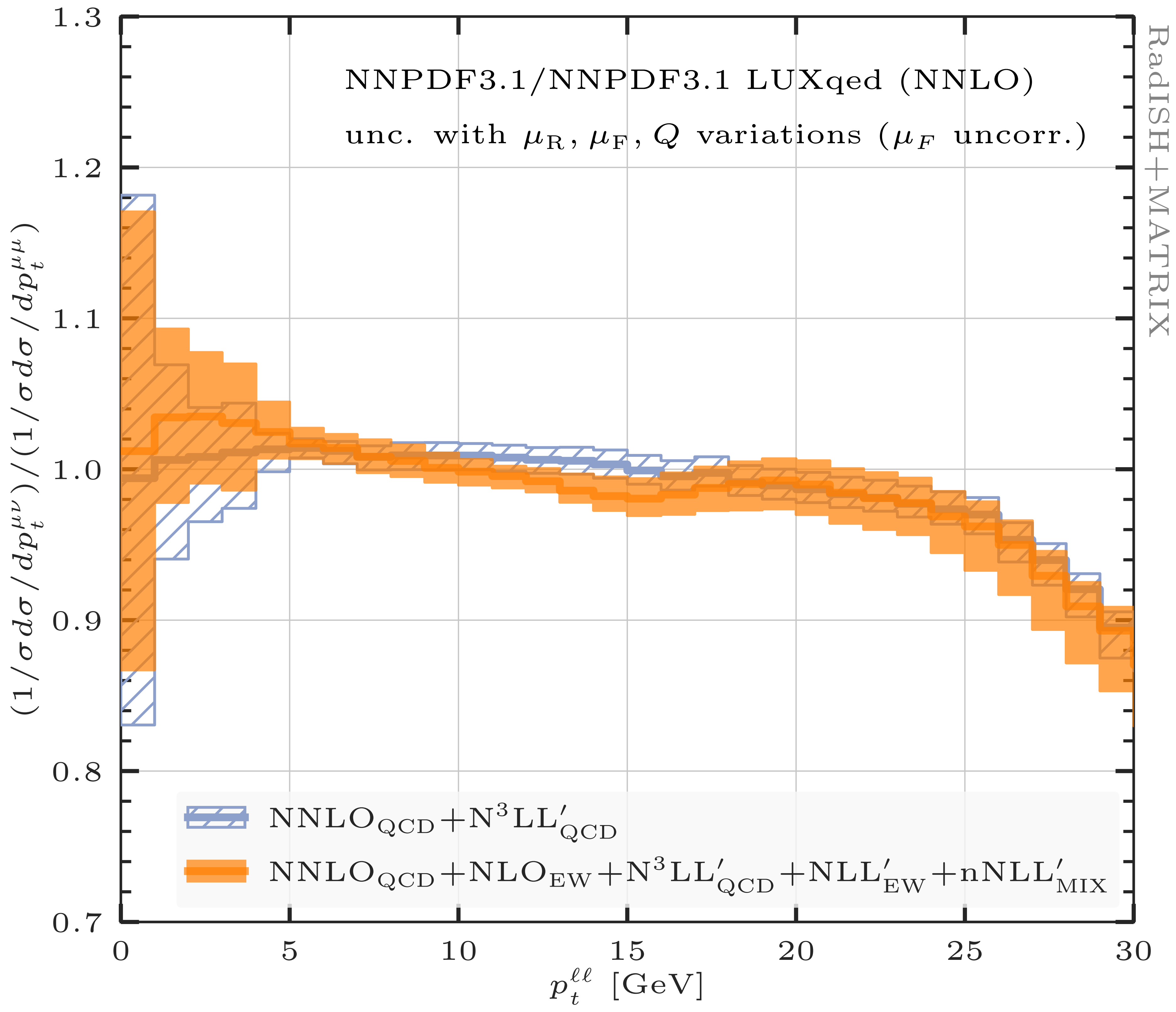
(Normalised) Ratio p_T^W / p_T^Z

Key ingredient in experimental strategies for measuring the W mass

- The impact of EW effects on the normalized ratio is mild but not negligible
- Under the assumption of correlated μ_R and uncorrelated μ_F variations, barely overlap within uncertainty bands

[Bizon,Gehrmann-De Ridder,Gehrmann,Glover,Huss,
Monni,Re,Rottoli,Walker '19]

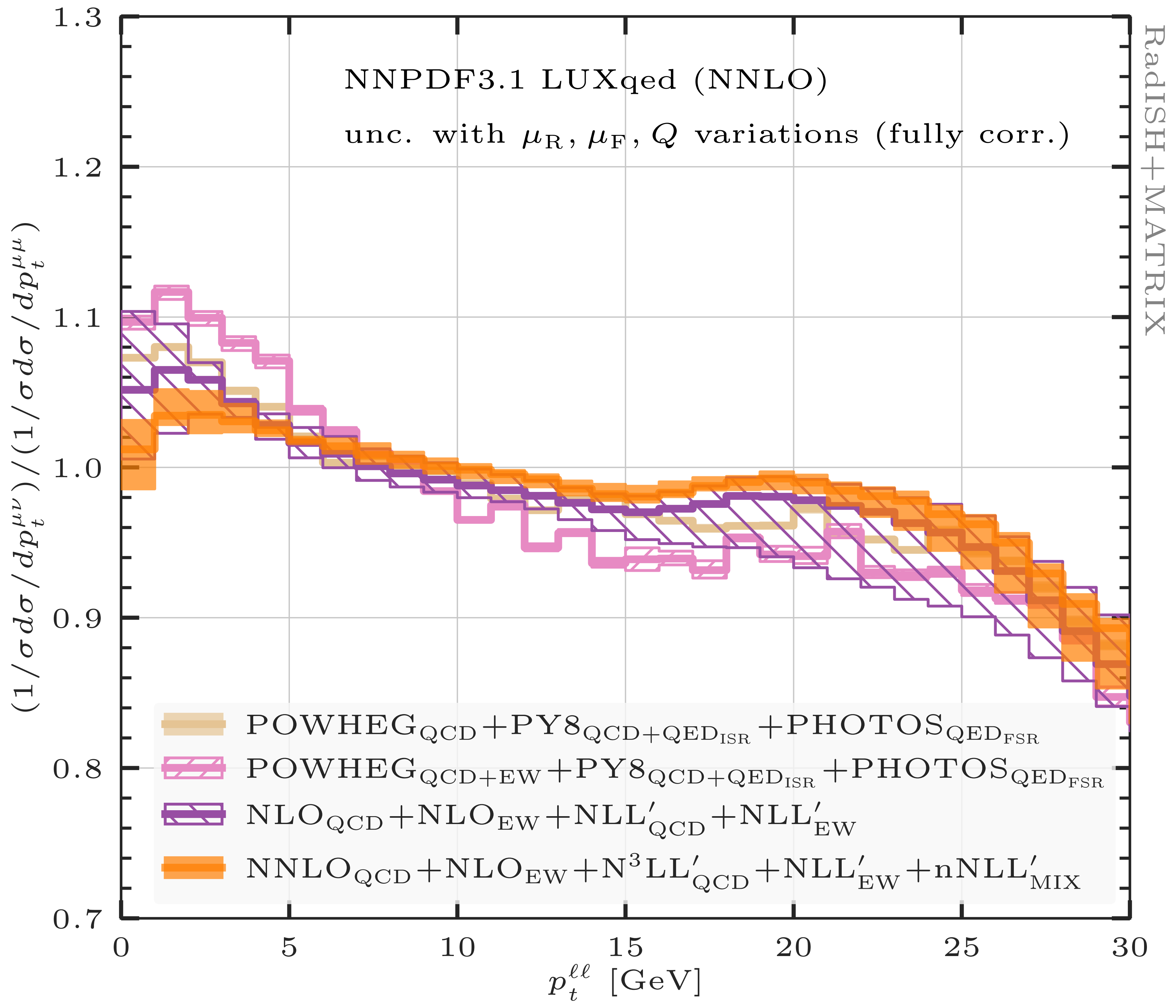


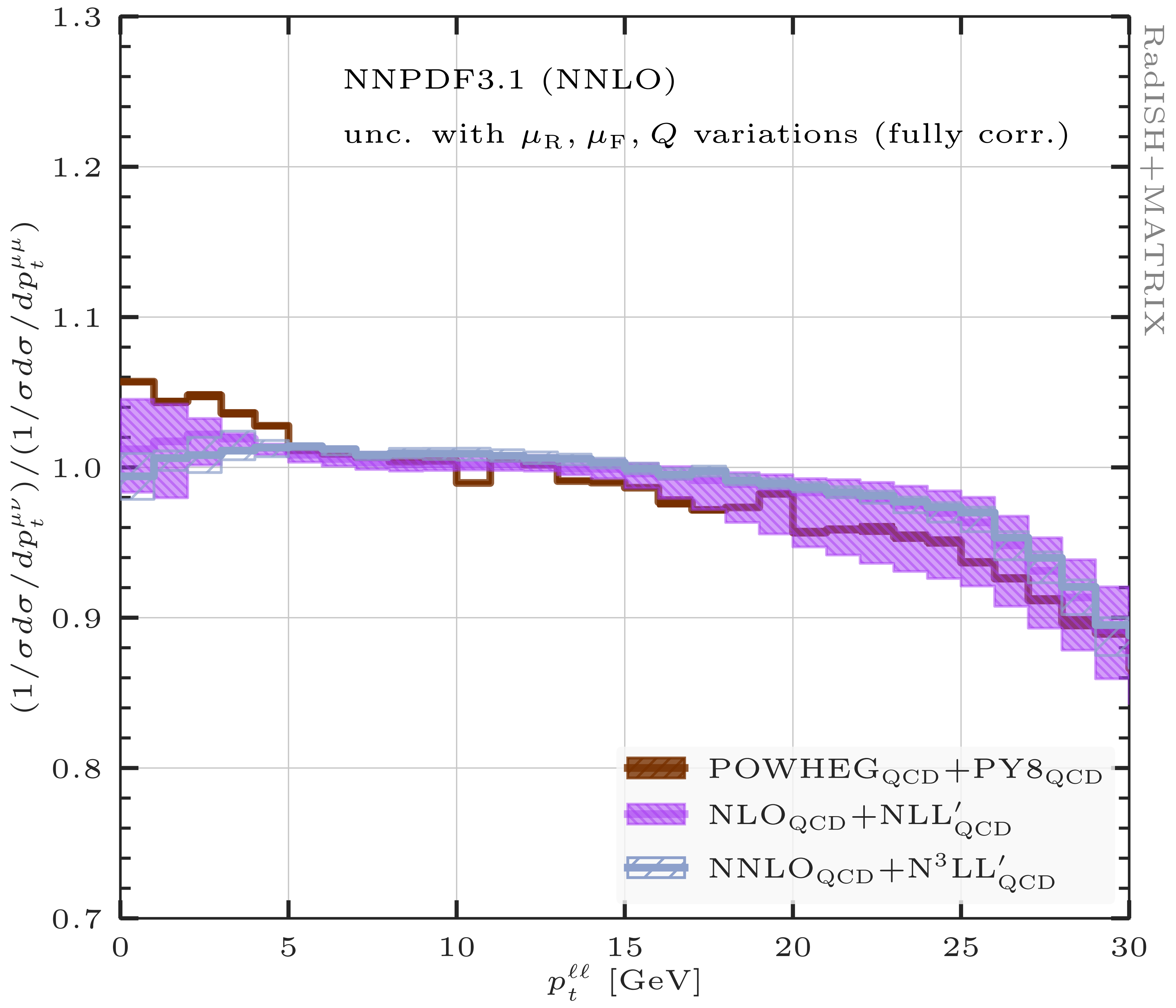


Ratio p_T^W / p_T^Z

Comparison with $\texttt{PWG}_{\text{EW}}\texttt{+PY8+PHOTOS}$, $\texttt{PWG}_{\text{QCD}}\texttt{+PY8+PHOTOS}$ and $\text{NLL}'_{\text{QCD}} + \text{NLO}_{\text{QCD}} + \text{NLL}'_{\text{EW}} + \text{NLO}_{\text{EW}}$

- Nice perturbative stability and robustness against shower tuning
- Better agreement of "simpler" $\texttt{PWG}_{\text{QCD}}\texttt{+PY8+PHOTOS}$ to RadISH, residual difference similar to pure QCD case
- $\texttt{PWG}_{\text{EW}}\texttt{+PY8+PHOTOS}$ result deviates significantly from our best prediction

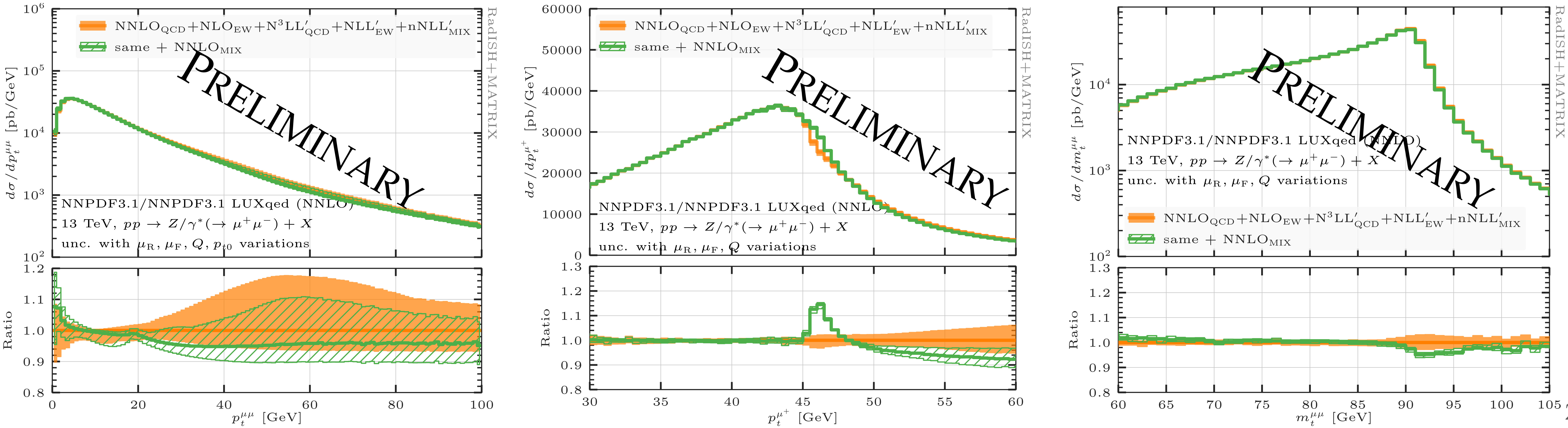




Caveat: impact of higher-order $\mathcal{O}(\alpha_s\alpha)$ matching

Non-singular $\mathcal{O}(\alpha_s\alpha)$ terms are enhanced by logarithms of the lepton mass!

- There is no apriori argument to believe that they are captured by standard procedures of error estimate (aka scale variation)
- They are needed to "match" what injected in the resummed component *See also talks by J. Mazzitelli and C. Biello*
- Personal thought: complete matching at $\mathcal{O}(\alpha_s\alpha)$ corrections should be sufficient for the moment (but very interesting to think about a refinement of the formalism to perform a joint resummation of lepton mass logarithms)



Summary and Outlook

- First combination of higher-order QCD resummation and the resummation of leading EW and mixed QCD-EW effects within the RadISH framework for **bare muons in CC and NC Drell-Yan**
- Definition of the observable: transverse momentum of a massive lepton pair in CC and NC Drell-Yan
- Impact on main DY observables: non negligible EW and mixed QCD-EW effects, mainly due to FSR QED
- First comparisons with state-of-art Monte Carlos
- Systematic analysis requires consistent matching of non-singular $\mathcal{O}(\alpha_s \alpha)$ corrections

Future prospects

- A more detailed phenomenological analysis (including matching and, possibly, non-perturbative corrections)
- Treatment of dressed electrons: requires to consider the resummation of a suitable observable
- Resummation of mass logarithms

Backup

Phenomenology impact on NC DY

Perturbative convergence at different QCD orders

- Overall good perturbative convergence
- As expected, effects of higher-order resummation are most visible at small $p_T^{\mu\mu}$ and near the Jacobian peak for p_T^{μ}, while the transverse mass is more stable against all-order radiation